Helmholtz Resonance

Glass bottle	1
Graduated cylinder	1
microphone	1
ruler	1
Vernier caliper	1
DataStudio	1

Blow over the opening of a bottle and you hear a tone. A pure tone is a sound wave with only one frequency, and it turns out that the tone made by a bottle is a fairly pure tone (mostly of a single frequency).

The bottle resonates a single frequency in what is known as Helmholtz Resonance. The air in the neck of the bottle is acting like an oscillating mass on a spring, and the spring is the compressible air in the main part of the bottle. We can calculate the resonant frequency if we can determine the oscillating mass and the spring constant, because the angular frequency of a mass on a spring is given by

$$\omega = \sqrt{k/m}$$

The spring constant of the air can be determined by treating it as an ideal gas that expands and contracts adiabatically. During the adiabatic oscillation, the pressure and volume in the bottle obey the following:

$$PV^{\gamma} = constant$$

γ is the adiabatic index of air. Taking the derivative of the above, we have:

$$P\gamma V^{\gamma-1}dV + dPV^{\gamma} = 0$$

Solving this for dP:

$$dP = -\frac{P\gamma V^{\gamma-1}dV}{V^{\gamma}}$$

$$dP = -\frac{P\gamma dV}{V}$$

dP is the change in the trapped gas's pressure from equilibrium. dV is the change in its volume from equilibrium. But $dV = xA$ where x is the distance that the air in the neck moves and A is the cross section area of the neck. Using this in the above expression:

$$dP = -\frac{P\gamma xA}{V}$$

The pressure change dP returns the oscillating mass to its equilibrium position. The total restoring force is:

$$F = AdP$$

We can combine this with the previous equation:

$$\frac{F}{A} = -\frac{P\gamma xA}{V}$$

$$F = -\frac{P\gamma A^2}{V}x$$

The last expression has the form of Hooke's Law with a spring constant $k = P\gamma A^2/V$. Thus we have the spring constant of the expanding and contracting air in the main part of the bottle. The angular frequency of the oscillation of the air mass in the neck is thus:

$$\omega = \sqrt{\frac{\gamma A^2 P}{mV}}$$

P is the ambient pressure of the air in the bottle, and so is one atmosphere.

Let's put this in a form that doesn't contain m, the mass of the air in the neck, but contains something more readily measured. To replace m, note that the mass of the air in the neck is the density of air multiplied by the volume of the neck

$$m = \rho A L$$

L is the length of the neck. We can also use the ideal gas law to find ρ.

$$PV = nRT$$

$$nM = m$$

The second equation notes that the mass of volume V of air is the number of moles times the molar mass of the gas. So then

$$PV = \frac{m}{M}RT$$

$$\frac{PM}{RT} = \frac{m}{V}$$

$$\frac{PM}{RT} = \rho$$

Combining this with the expression for the mass of the air in the neck

$$m = \frac{PMAL}{RT}$$

And putting this into the frequency equation, we have

$$\omega = \sqrt{\frac{\gamma A^2 P}{V}\frac{RT}{PMAL}}$$

$$\omega = \sqrt{\frac{\gamma ART}{MLV}}$$

The cyclic frequency is a bit more useful than the angular frequency, since this is what we'll measure.

$$f = \frac{1}{2\pi}\sqrt{\frac{\gamma ART}{MLV}}$$

A is the cross section area of the neck, R is the universal gas constant, T the Kelvin temperature of the air, M the molar mass of air, L the length of the neck, and V the volume of the bottle.

Procedure

Make sure to record everything in SI units.

1. Measure the inner diameter of the neck and use this to find its cross section area. $A =$ _______
2. The kelvin temperature in the room is the celcius temperature plus 273.15 $T =$ _______
3. Air is composed mostly of diatomic molecules. Find the adiabatic index of diatomic gases. $\gamma =$ _______
4. Find the molar mass of air. $M =$ _______
5. Measure the length of the neck of the bottle $L =$ _______
6. The bottle's volume V is listed on the label. Convert it to SI units and record it here. $V =$ _______
7. Now find f for the bottle $f =$ _______
8. We'll do the experiment twice more, with $250\ ml$ of water in the bottle, and with $500\ ml$ of water in the bottle. Calculate ω again for the two new volumes ($V - 250ml$ and $V - 500ml$).

V	f_{theory}	$f_{measured}$	$\%\ diff$

Measuring the Frequencies Produced by the Bottle

There are several ways to measure the frequencies that the bottle produces. You can use this online tool. When you play the tone that the bottle makes, the webpage should read out the frequency. If it doesn't, you might have to click on the link above where it says "Make sure you are using the address…."

You can also download an app for your phone that measures musical frequencies. One free app that you can put on your phone is called Pitched Tuner. The icon is shown here for reference. When the app is started, you simply blow the bottle to make the tone, and it will display the measured frequency.