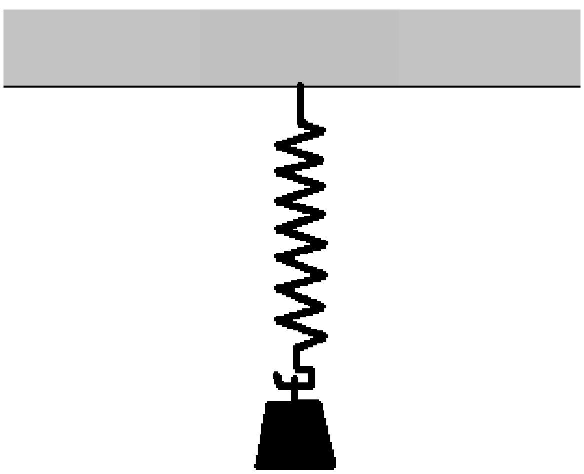
The Simple Harmonic Oscillator

Equipment	Qty.
Triple Beam Balance	1
Pendulum Clamp	1
Meter Stick	1
Table Clamp & Rod	1
S.H.M. Container	1
Measuring Equipment Tray	1

Many things in nature oscillate, and so a quantitative understanding of oscillatory behavior is important.

A simple example that illustrates oscillating systems is an object oscillating on a spring, shown below.



Most springs have a linear response, meaning that when stretched, the **restoring force** (the force tending to return it to its un-stretched length) is proportional to the distance that it's stretched.

Determining the Spring Coefficient

First you'll directly determine the spring constant of your spring by stretching it with known amounts of force (hanging weights) and measuring the distance that it stretches.

Procedure

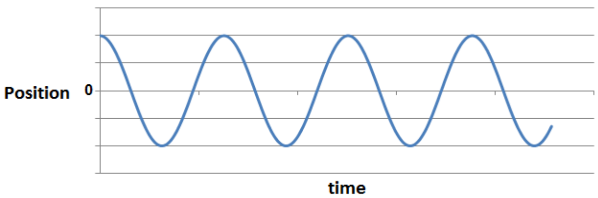
1. Mount the table clamp to the table and the long rod in the clamp. Mount the pendulum
2. clamp high on the rod and hang the spring from the end knob.
   Suspend the mass hanger from the bottom
3. end of the spring.
4. With no masses on the spring, we'll call this zero displacement and zero force. Measure the height of the bottom of the spring from the floor.
5. Place the mass hanger and measure the height again.
6. Add one 50g mass on the hanger and measure the height again.
7. Repeat until the table is complete.
8. In each row, the force on the spring is the weight of the mass. the displacement x is the distance from the floor subtracted from the distance with zero mass.
9. Now graph force (y axis) as a function of x (x axis).
10. Find the slope of the graph. This is

    the spring constant. $k =$ ______

Mass (grams)	Force (N)	Distance From floor	x (m)
0			0
50			
100			
150			
200			
250			
300			

Measuring the Oscillation Period and Determination of *k* and the Effect of the Mass of the Spring.

Oscillation that's caused by a linear restoring force is called **Simple Harmonic Motion**. If you were to plot the object's position as a function of time, the graph would be sinusoidal, like the graph below.



We can derive an equation of motion for this system. To start, note that the restoring force in the spring provides the vertical accelerating force for the mass.

$$F = -kx$$

Putting this force into Newton's Second Law of Motion we have:

$$-kx = ma$$

$$-kx = m\frac{d^2x}{dt^2}$$

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x$$

A solution to this differential equation is:

$$x = A\sin\left(\sqrt{k/m}\,t\right)$$

So the mass oscillates sinusoidally. Since the sine function completes a single cycle when the expression in parentheses increases by 2π, the period of oscillation can be determined:

$$2\pi = \sqrt{k/m}\,T$$

$$T = 2\pi\sqrt{\frac{m}{k}}$$

But the amount of mass that is oscillating is more than m, since the spring is also moving. Not the entire spring is oscillating with the object though, because the upper part isn't moving as fast as the lower part. So some fraction of the spring's mass (we'll call it ΥM where M is the spring's mass) should be added to the mass of the object in order to model the system properly.

$$T = 2\pi\sqrt{\frac{m + \Upsilon M}{k}}$$

We don't know what fraction f to add, but we can find it experimentally as we also find the spring constant. To do this, let's put the above equation into a different form. Start by squaring both sides.

$$T^2 = (2\pi)^2\left(\frac{m + \Upsilon M}{k}\right)$$

$$T^2 = \frac{(2\pi)^2}{k}(m + \Upsilon M)$$

$$T^2 = \frac{(2\pi)^2}{k}m + \frac{(2\pi)^2}{k}\Upsilon M$$

Note that the equation now has the form of a straight line when T^2 is graphed as a function of m. The $slope$ and $y\ intercept$ are:

$$slope = \frac{(2\pi)^2}{k}$$

$$y\ intercept = \frac{(2\pi)^2}{k}\Upsilon M$$

Thus these two values, determined from the graph, can be used to determine k and Υ. You'll compare this determination of k with the one in the previous section.

Procedure:

In this part, you *will* include the mass of the mass hanger in the mass column.

1. Measure the mass of the spring.
   $M =$ _______
2. Suspend the mass hanger and one 100 g mass from the spring.
3. Time this system for 20 complete oscillations and calculate T and T^2 from this.
4. Add a 50 g mass and repeat step 3.
5. Repeat steps 3 and 4 until m
6. Graph T^2 (y axis) as a function of m in kg (x axis).
7. Determine the slope of the graph.
   $\text{slope} =$ _______
8. Determine the y intercept of the graph
   $y\ intercept =$ _______
9. Find k from the slope. $k =$ _______
10. Find the percent difference between this k and the one determined in the first procedure. $\%\ diff =$ _______
11. Use the new k, the $y\ intercept$, and the spring mass M to calculate Υ.
    $\Upsilon =$ _______

m	t for 20 oscillations	T	T^2
150 g			