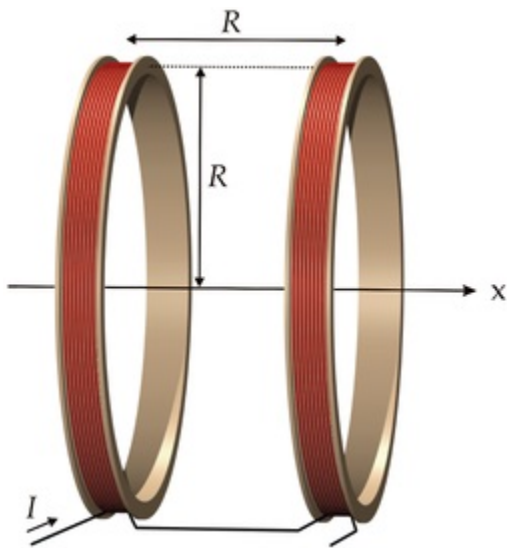


The Magnetic Field

Equipment	Quan
Helmholtz Coil	1
Magnetic Field Sensor	1
Low Voltage Power Supply	1
Multimeter	1
Patch wires	mult
Alligator clips	mult

Electric current produces a magnetic field. In our experiment, the field is generated by a Helmholtz Coil, shown below. We'll measure the field produced at the center of the coil to see how it compares to a theoretical prediction of the field strength.

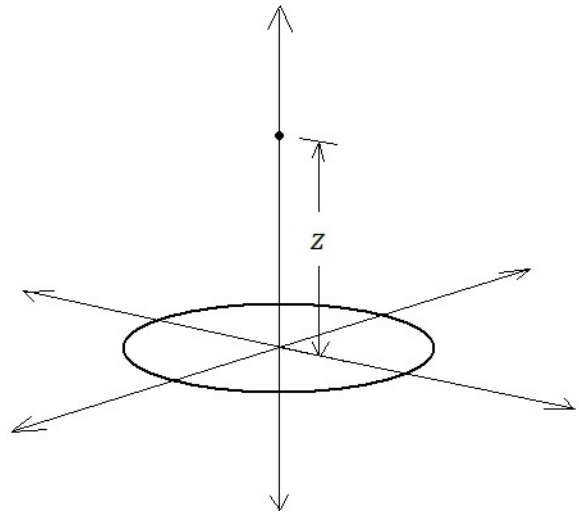


A Helmholtz coil produces a relatively homogeneous magnetic field in a small region near its center. We can derive an equation for the magnitude of this field.

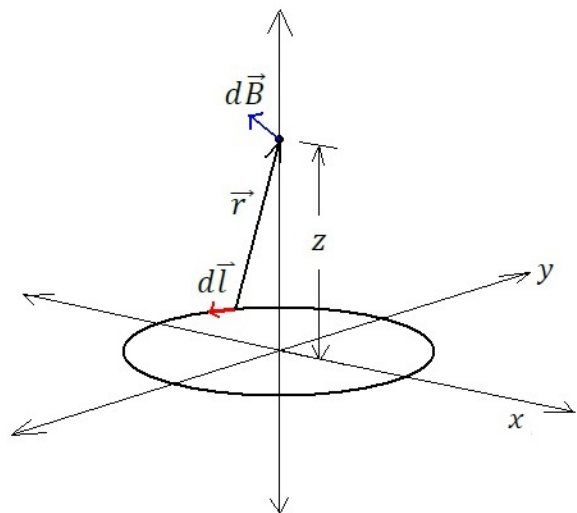
To start the derivation, we note that an infinitesimal wire element $d\vec{l}$ produces an infinitesimal field element $d\vec{B}$ at a position \vec{r} from $d\vec{l}$. This element of magnetic field is given by:

$$d\vec{B} = \frac{\mu_0 I d\vec{l} \times \vec{r}}{4\pi |\vec{r}|^3}$$

Using this, we'll find the magnitude of the field at a point on the axis of a wire loop (below) at a distance z from the plane of the loop.

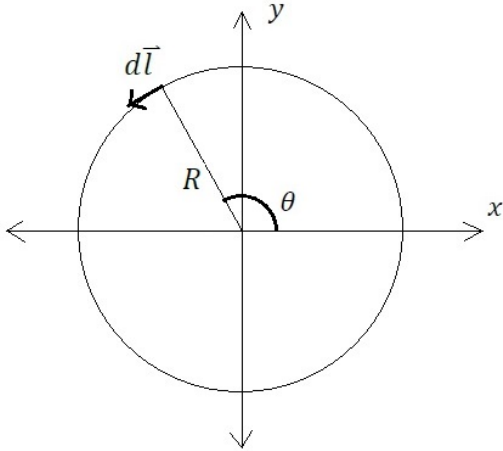


The infinitesimal contribution to the field of the wire length element $d\vec{l}$ is shown in the figure below. It is perpendicular to both $d\vec{l}$ and \vec{r} .



It can be seen in the figure below that infinitesimal element $d\vec{l}$ can be expressed as follows:

$$d\vec{l} = -\sin\theta R d\theta \hat{i} + \cos\theta R d\theta \hat{j}$$



And it can be seen from the previous figure that the position vector \vec{r} can be expressed as:

$$\vec{r} = -\cos\theta R \hat{i} - \sin\theta R \hat{j} + z \hat{k}$$

Thus $d\vec{l} \times \vec{r}$ can be written:

$$d\vec{l} \times \vec{r} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -\sin\theta R d\theta & \cos\theta R d\theta & 0 \\ -\cos\theta R & -\sin\theta R & z \end{vmatrix}$$

It is apparent from the symmetry of the problem that only B_z will survive the integration of the field. The other two components integrate to zero. Thus we can ignore all but the z component of the cross product.

$$(d\vec{l} \times \vec{r})_z = (\sin^2\theta R^2 d\theta + \cos^2\theta R^2 d\theta) \hat{k}$$

$$(d\vec{l} \times \vec{r})_z = R^2 d\theta \hat{k}$$

The expression for $d\vec{B}_z$ also contains $|\vec{r}|$, which is equal to:

$$|\vec{r}| = \sqrt{z^2 + R^2}$$

Thus dB_z is equal to:

$$dB_z = \frac{\mu_o I R^2 d\theta}{4\pi(z^2 + R^2)^{3/2}}$$

Integrating around the loop:

$$B_z = \int_0^{2\pi} dB_z$$

$$B_z = \frac{\mu_o I R^2}{4\pi(z^2 + R^2)^{3/2}} \int_0^{2\pi} d\theta$$

$$B_z = \frac{\mu_o I R^2}{2(z^2 + R^2)^{3/2}}$$

This is the field at a point on the axis due to a single loop of wire carrying current I , so the field due to 2 coils of N loops would be this equation times $2N$. And since each coil is a distance $R/2$ from the center point (the point where the field will be measured) then $z = R/2$. Thus the total field due to the Helmholtz coil is:

$$B_z = \frac{2\mu_o I R^2 N}{2((R/2)^2 + R^2)^{3/2}}$$





$$B_z = \frac{\mu_o I R^2 N}{(\frac{1}{4}R^2 + R^2)^{3/2}}$$

$$B_z = \frac{\mu_o I N}{R(5/4)^{3/2}}$$

$$B_z = \frac{8\mu_o I N}{5\sqrt{5}R}$$

You'll measure the field strength to see if it indeed obeys this equation. In our apparatus, $N = 100$.

Procedure

1. Plug the magnetic field sensor into Analog Channel A of the 850 Universal Interface box (it's next to your table's computer).
2. To start the Capstone software, double click on the Capstone icon on the computer screen. It might take 10 or 15 seconds for the program to start, so be patient. If you double click again, you might start the program twice, which can cause it to not work properly. 
3. Click on the "Hardware Setup" icon. 
4. You should see an image of the data collection box. Click on Analog Input A.
5. Choose "Magnetic Field Sensor."
6. On the righthand side of the screen, double click on the "Digits" icon. 
7. Click on <Select Measurement> and choose "Magnetic Field Strength (1X) (T)."
8. Since our field is relatively weak, we need to change the readout to scientific notation mode. Click the properties icon, then on "Numerical Format," and change "Number Style" to scientific notation. 
9. The Record button should now be red. Click it and then press the tare button on the magnetic field sensor. The digital readout should show a small field strength.
10. Measure the diameter of one of the Helmholtz coils, and from this, find R , its radius. Since each coil consists of many loops with slightly different radii,

measure the diameter of the bundle of loops from the middle of the bundle.

$$R = \underline{\hspace{2cm}} m$$

11. Wire the setup as in the schematic. Note that the current must travel in the same circular direction in each coil.
12. Before turning on the power, turn the DC voltage knob all the way to the left and the DC current knob all the way to the right.
13. Note that the magnetic field produced by the coil will be parallel to the axis of the loops. Study the magnetic field sensor to determine how best to use it in order to measure the field.
14. Set the sensitivity of the field sensor on 1X.
15. Position the sensor in the very center of the Helmholtz coil. Before powering up the coil, click "Record" and then press "tare" on the sensor. This calibrates the sensor to zero.
16. Turn on the power supply and measure current and resulting magnetic field strength for 5 different currents. The voltage knob is used to adjust the current.
17. Use the equation for B_z to find the theoretical field for each of currents you used.
18. Calculate percent errors for your measurements.

I (A)	B_z (T)	B_z calc	% err

The Magnetic Force

Equipment	Quan
Apparatus with Small Coil	1
Sliders	2

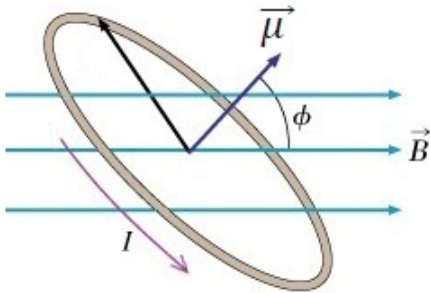
Magnetic Force On the Smaller Coil

$$B_z = \frac{8\mu_o IN}{5\sqrt{5}R}$$

In the next part of the experiment, the magnetic field generated by the Helmholtz coil will exert a torque on a smaller coil that pivots at its center.

The torque on a loop of wire in a magnetic field is given by:

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$



$\vec{\mu}$ is the magnetic moment of the loop. For a planar wire coil of M loops with current I_S and loop area vector \vec{A} , $\vec{\mu}$ is given by:

$$\vec{\mu} = I_S M \vec{A}$$

So then:

$$\vec{\tau} = I_S M \vec{A} \times \vec{B}$$

In our case, the small coil's plane is parallel to the field lines, which means $\vec{A} \perp \vec{B}$, and so $|\vec{\tau}|$ is:

$$\tau = I_S M A B$$

In our experiment, the same current will flow through both the Helmholtz Coil and the smaller coil, and so $I_S = I$, and the torque induced by our field is:

$$\tau = I M A \frac{8\mu_o I N}{5\sqrt{5}R}$$

$$\tau = \frac{8I^2 \mu_o M N A}{5\sqrt{5}R}$$

This can be written as:

$$\tau = \frac{8\mu_o M N A}{5\sqrt{5}R} I^2$$

So according to our derivation, the torque exerted on the smaller coil is proportional to the square of the current in the device. This means that if we run 3 current levels through the device and measure the torque in each case, then plot τ as a function of I^2 , the 3 points should fit a straight line, and the slope of the line should be:

$$\text{slope} = \frac{8\mu_o M N A}{5\sqrt{5}R}$$

You'll plot this graph, measure the slope, then using this slope and the values for M , N , A , & R , you'll calculate μ_o . You'll then compare this to the accepted value, which is $\mu_o = 4\pi \times 10^{-7} \text{ N/A}^2$. In our apparatus, $M = 50$.

Procedure

1. Measure the diameter of the smaller coil as you did for the larger coil, and record the radius R_S . Remember to measure the diameter from the center of one side of the wire bundle to the other.

$$R_S = \text{_____}m$$

2. Calculate the area of the small coil.

$$A = \text{_____}m^2$$

3. Your instructor will show you how to connect the power supply to the device.
4. Use the digital scale to find the mass of the plastic slider.

$$m_{sl} = \text{_____}kg$$

5. Balance the device so that the small coil is roughly horizontal. An aluminum slide is used to do this. Then move the magnet up or down on the rod to align it with the L-shaped beam. This will be the "pointer".
6. Before turning on the power, turn the DC voltage knob all the way to the left and the DC current knob all the way to the right.
7. Place the plastic slider at the 2.0 cm position on the L-shaped beam.
8. Turn on the power supply and increase the voltage until the device returns to balance (the beam comes back to the pointer). Record the current I and $r = 0.020 m$ in the table below. (IF THE CURRENT CAUSES THE BEAM TO TIP THE SAME WAY AS THE SLIDER, THEN SWAP THE TWO WIRES AT THE POWER SUPPLY TO REVERSE THE EFFECT).
9. Move the plastic slider to the 4.0 cm position.
10. Adjust the current until the device is rebalanced. Record I & r .
11. Repeat for 6.0 cm & 8.0 cm
12. TURN OFF THE CURRENT SO THAT THE APPARATUS DOESN'T OVERHEAT.

13. For each value of the slider position, calculate the torque τ that the slider's weight exerts around the axis and enter the values into the table (REMEMBER, YOU'RE NOT CALCULATING THE MAGNETIC TORQUE FROM THE DERIVED EQUATION, BUT THE MECHANICAL TORQUE OF THE SLIDER).

14. Calculate the square of each of the currents and record them in the table.

15. Plot a graph of τ as a function of I^2 .

16. Find the slope of this plot.

$$slope = \text{_____}Nm/A^2$$

17. According to our torque equation, the slope should be equal to

$$slope = \frac{8\mu_o MNA}{5\sqrt{5}R}$$

Using the value of the slope from your graph and the values of $M, N, A, \& R$, calculate the value of μ_o . Note that R is the radius of the Helmholtz coil, not the small coil, and A is the area of the small coil.

$$\mu_o = \text{_____}N/A^2$$

18. Find the percent error of the value that you determined for μ_o .

$$\%error = \text{_____}\%$$

I (A)	r (m)	I^2 (A ²)	τ (Nm)