Inductors & Self-Inductance

Equipment	Qty.
Low Voltage Power Supply	1
LCR meter	1
BB Wires	2
Alligator Clips	2
Magnetic Field Sensor	1

Electric current in a conductor produces a magnetic field. When the current changes, the field changes, and this causes an electromotive force according to Faraday's Law. For a coil of N loops of wire, this is…

$$\mathcal{E} = -N\frac{\partial \Phi}{\partial t}$$

Φ is the magnetic flux generated by the current, and the minus sign in the equation means that the emf opposes the change that is causing the magnetic field to change. Thus a changing current produces an electromotive force that tends to slow the change in the current. This is known as **self-inductance**, and it endows electric current with a kind of inertia, tending to resist a change in the current.

How much emf a circuit produces from a given rate of current change depends on the circuit's spatial configuration, and the **inductance** L of a circuit is the constant of proportionality relating the rate of change of current to the emf produced.

$$\mathcal{E} = -L\frac{\partial I}{\partial t}$$

Combining this with Faraday's Law above:

$$L\frac{\partial I}{\partial t} = N\frac{\partial \Phi}{\partial t}$$

$$LI = N\Phi$$

$$L = \frac{N\Phi}{I}$$

We'll consider this the definition of a coil's inductance.

The electromotive force also drives current in nearby conductors, and this is known as **mutual inductance**, but we'll mostly be concerned with self-inductance here, and L is a measure of this.

A circuit that produces a large magnetic field (e.g. a solenoid) will also have a large inductance. The approximate inductance of a solenoid can be derived from an approximate expression for the magnetic field that it generates

$$B = \frac{\mu_o NI}{\ell}$$

Since B is the flux density, the total flux through the coil is

$$\Phi = \frac{\mu_o NIA}{\ell}$$

A is the cross section area inside the solenoid, and since $A = \pi R^2$ then

$$\Phi = \frac{\mu_o NI\pi R^2}{\ell}$$

Combining this with our inductance definition, we obtain:

$$L = \frac{\mu_o \pi R^2 N^2}{\ell}$$

R is the radius of the solenoid, ℓ is its length, and N is the number of loops.

Measuring Inductance in Coils

First you'll measure the inductance of some coils with the inductance meter and compare the values to those predicted by the above expression.

1. For 3 coils, measure the loop radius R, coil length ℓ and number of loops N. Also record each coil number.
2. Calculate the inductance L of each coil using the equation above.
3. Measure each inductance using the meter. [Note: the meter has two modes for measuring inductance, which are set using the SER/PAL button. Make sure that the meter is in the SER mode]
4. Find the percent error for each inductance value.

Coil #	R	ℓ	N	L calc	L meas	% error

Inductors in Parallel and in Series

The circuit below shows two coils in series with inductances L_1 and L_2 and a current I that flows through the circuit. Each coil generates its own emf when the current changes
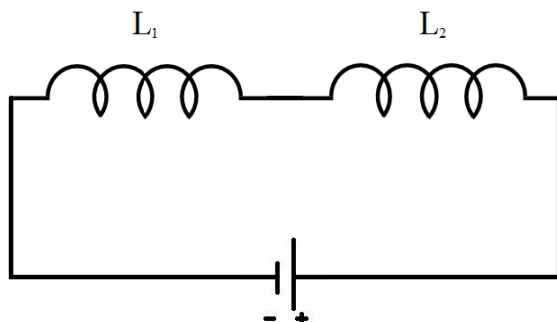
$$\mathcal{E}_1 = L_1 \frac{dI}{dt}$$

$$\mathcal{E}_2 = L_2 \frac{dI}{dt}$$

Since they are in series, the total emf due to the two inductors is $\mathcal{E}_{tot} = \mathcal{E}_1 + \mathcal{E}_2$, and so
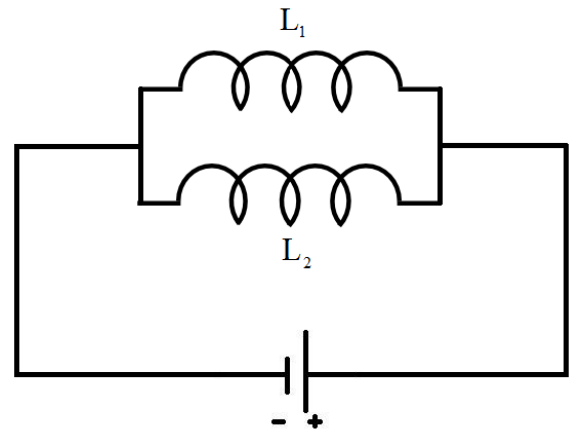
$$\mathcal{E}_{tot} = (L_1 + L_2)\frac{dI}{dt}$$

This equation is like the two previous equations, but treating the pair of inductors as a single inductor with inductance

$$L_{tot} = L_1 + L_2.$$

Thus for two inductors in series, the total equivalent inductance is additive.

On the other hand, for two inductors in parallel as in the figure below, the emf across each is the same, but the current might be different

$$\mathcal{E} = L_1 \frac{dI_1}{dt}$$

$$\mathcal{E} = L_2 \frac{dI_2}{dt}$$

But the total current in the circuit splits between the two parallel inductors so that $I_{tot} = I_1 + I_2$. Taking the derivative of this with respect to time we have

$$\frac{dI_{tot}}{dt} = \frac{dI_1}{dt} + \frac{dI_2}{dt}$$

Combining this with the two previous expressions, we see that

$$\frac{dI_{tot}}{dt} = \frac{\mathcal{E}}{L_1} + \frac{\mathcal{E}}{L_2}$$

$$\frac{dI_{tot}}{dt} = \mathcal{E}\left(\frac{1}{L_1} + \frac{1}{L_2}\right)$$

$$\mathcal{E} = \left(\frac{1}{L_1} + \frac{1}{L_2}\right)^{-1}\frac{dI_{tot}}{dt}$$

This last equation treats the pair of parallel inductors as a single inductor with inductance given by

$$\frac{1}{L_{tot}} = \frac{1}{L_1} + \frac{1}{L_2}$$

These two rules for adding inductances for series and parallel combinations may remind you of similar rules for finding total resistance of resistors in series and parallel.

Now you'll test these rules by measuring the total inductance of both kinds of combinations.

1. Connect in series two coils whose inductances you've already measured.
   $L_1 = $ _____ $L_2 = $ _____
2. Measure the inductance of this combination. $L_{ser} = $ _____
3. Connect the same two coils together in parallel.
4. Measure the inductance of this combination. $L_{par} = $ _____
5. Calculate the theoretical inductances of the series and the parallel combinations.
 $L_{ser} = $ _____ $L_{par} = $ _____

6. Find the percent error for each of your measured inductances.

Generating Magnetic Fields with the coils

Circuits with large inductances typically generate large magnetic fields per amp of current. For example, combining the expressions above for the inductance and the magnetic field of a solenoid, we see that:

$$B = \frac{LI}{\pi R^2 N}$$

So then the field produced by a solenoid is proportional to its inductance.

1. Plug the magnetic field sensor into the Analog port A of the Capstone Interface.
2. Double click on the Capstone icon on the desktop.
3. Click the "Hardware Setup" icon.
4. You should see an image of the data collection box. Click on Analog Port A and choose "magnetic field sensor."
5. Click "Hardware Setup" again to dismiss it.
6. On the righthand side of the screen, double click on "Digits."
7. A digital readout should appear. Click on <Select Measurement> and choose "Magnetic Field Strength (100X) (T)."
8. The Record button should now be red. Click it and then push the "tare" button on the magnetic field sensor. The digital readout should show a small magnetic field strength. The tare button can be used to zero out

the reading at any time, which may be necessary if the sensor drifts.

9. Wire one of the solenoids in series with the low voltage power supply.
10. Insert the narrow end of the magnetic field sensor into the solenoid so that the tip is well inside the solenoid's end.
11. Before turning on the power, turn the DC voltage knob all the way to the left and the current knob all the way to the right.
12. Turn on the power and start the magnetic field measurement.
13. Slowly turn up the voltage until the current readout on the power supply reads 5 amps.
14. Record the magnetic field in the center of the solenoid.
15. Repeat for the other solenoid.
16. Compute the predicted field strengths for these solenoids from the equation on page 1.
17. Compare the measured field with the theoretical field by calculating the percent error.

Coil #	I	B_{meas}	B_{theo}	% err