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Conservation of Energy & Rotational Dynamics

Experimental Objectives

The objective of this experiment is to study the law of conservation of energy and some concepts of rotational dynamics. We will apply the relationship between the potential energy and kinetic energy of a spherical object rolling (without slipping) down a ramp (inclined plane) to determine the velocity with which the object leaves the end of the ramp. The object hits the floor following a free fall trajectory from the ramp. The horizontal displacement of the object, measured from a point below the ramp, will then be used to predict the velocity with which the object leaves the ramp.

Overview

In this experiment, a metal ball will be released from a vertical height h on a ramp inclined at an angle θ below the horizontal so that it rolls down the ramp without sliding. At the end of the ramp, the ball leaves with a velocity v and falls freely under the effect of the force of gravity before hitting the floor at a point at some horizontal distance R from the edge of the table. Considering the relatively low speed of this projectile motion, we will ignore the effect of air resistance on the flight of the ball.

Three values of the velocity of the ball at the end of the ramp will be compared. The theoretical velocity $\mathbf{v_t}$ will be determined by considering the conservation of energy as the ball rolls from its point of release to the end of the ramp. The ballistic velocity $\mathbf{v_b}$ of the ball will be determined from the measured values of the horizontal displacement of the ball from the end of the ramp to the point of impact on the floor. The measured velocity $\mathbf{v_m}$ will be determined directly using a Photogate assembly.

The Law of Conservation of Energy states that the total mechanical energy of an isolated system is constant. Thus, in the absence of external forces acting on the system, the sum of the kinetic energy K and potential energy U must remain constant. Therefore,

$$K_{final} + U_{final} = K_{initial} + U_{initial}$$

In order to calculate the theoretical velocity, we set the total mechanical energy at the point of release on the ramp equal to the total mechanical energy at the bottom of the ramp. At the bottom of the ramp where the ball leaves the ramp, the total kinetic energy (K) of the ball is the sum of its translational kinetic energy and rotational kinetic energy. Thus:

$$K = \tfrac{1}{2} m v_{cm}^{2} + \tfrac{1}{2} I_{cm} \omega^2$$

where v_{cm} = the magnitude of the velocity of the center of mass of the ball and I_{cm} = the moment of inertia of the ball about its center of mass.

The ballistic speed of the ball will be determined by considering the two-dimensional projectile motion. Since the ball leaves the ramp at some angle below the horizontal, the velocity must be resolved into horizontal and vertical components. The derivation of the equation for determining the relationship between the horizontal displacement (*R*), vertical displacement (*Y*) from the bottom edge of the ramp to the floor, and the ballistic velocity $\mathbf{v_b}$ will be left as an exercise for the student.

Apparatus

Photogate assembly	**Balance**
Plumb bob with string	**Inclinometer**
2-meter ruler and Calipers	**Rods and double 90° clamps**
Carbon paper, white paper	**Universal Table Clamp**
Masking tape (roll)	**Three-finger clamp**
Grooved ball track (ramp)	**Three-legged base ring stand**

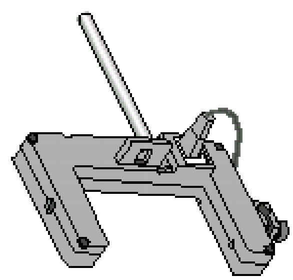
PROCEDURE:

P1: Computer/Photogate Setup

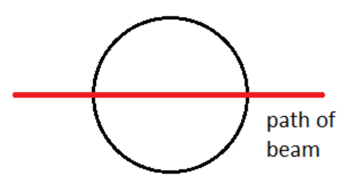
1. The *850 Universal Interface* and the computer should be already switched on.

2. Connect the Photogate's phone plug to Digital Channel 1 of the interface.

3. Using the three-legged base ring stand and suitable rods and clamps, suspend the U-shaped photogate above the path (with the "U" facing downward) the ball takes just as it leaves the ramp. Ideally, the Photogate should be set up perpendicular to the ramp. The ball should pass through the photogate, blocking and un-blocking the light beam of the photogate as it passes without hitting the photogate. Be careful not to change this vertical position of the photogate after the initial setup.



4. It's important to arrange the photogate so that as the ball passes, the beam is broken by the whole diameter (figure at right). The software will measure the time that the beam is broken, and then use the ball's diameter to calculate speed.



5. Double click the PASCO Capstone icon to start the software

6. Click on the "Hardware Setup" icon. You should see an image of the 850 box. Click on port 1 in the image. Choose "Photogate." Click "Hardware Setup" again to dismiss it.
7. Click the "Timer Setup" icon. Since we'll be using a preconfigured timer, click "Next."
8. Click the "Select a Timer" dropdown menu and choose "One Photogate (Single Flag).
9. In the next step, just the Speed box should be checked. Then click "Next."
10. Set the flag width to 0.0254 m. This is one inch, the ball's diameter. Click "Next."
11. Click "Finish." Click again on the "Timer Setup" icon to dismiss it.
12. The "Record" button should now be red, indicating that the photogate can be used.

P2: Equipment Setup and Data Acquisition

1. With the large rod clamp, attach the long rod on the long side of the table, near the shorter edge of the lab table. Attach the grooved ball wooden track (ramp) to be used as the ramp to the long rod, with the concave side upward, using the three-finger clamp.
2. By adjusting the appropriate clamps, set the angle of inclination of the ramp such that it falls in the range between 10° and 15°. Set the bottom edge of the ramp close to the edge of the table so that the rolling ball does not hit the edge as it leaves the ramp.
3. Carefully secure the end of the ramp firmly to the table with the masking tape so that the ramp does not move and ensure that no objects interfere with the motion of the ball as it leaves the ramp. You will need consistent setup conditions for the five data runs of the ball down this ramp.
4. The photogate can then be mounted for measurement of the ball speed at the bottom of the ramp. Remember that the beam should cut across the ball's center.

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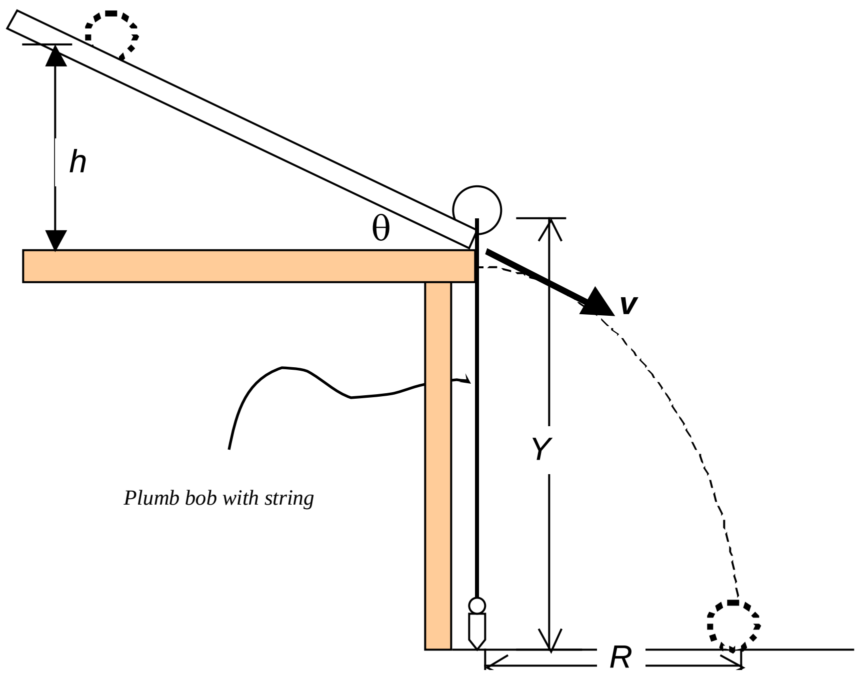
SAFETY REMINDER • **DO NOT let the ball hit anyone. Place a suitable backstop to "catch" the ball after it hits the floor.** • **Follow directions for using equipment.**	**THINK SAFETY** **ACT SAFELY** **BE SAFE!**



Experimental setup with the ball rolling down the ramp

5. Measure the length of the distance traveled by the ball along the ramp, from a pre-set point of release to the edge of the ramp. Record this measurement on the Data Sheet.

6. Measure the vertical height h_1 of the center of the ball above the table at the release point. Measure height h_2 of the center of the ball above the table at the end of the

ramp. Measure the value of the angle θ measured using the inclinometer. Record these measurements on the Data Sheet.

7. Hang the plumb bob on a string passing through the point where the ball leaves the ramp. Lower the plumb bob until it almost touches the floor. Place a piece of masking tape on the floor directly under the plumb bob and mark this position on the floor as the reference point for measuring the horizontal displacement of the ball to the point of impact on the floor.

8. Use the 2-meter stick to measure the vertical height of the ball (Y) above the floor at the point where the ball leaves the ramp. Measure to the bottom (not the center) of the ball since it is the bottom of the ball that strikes the floor. Record this measurement on the Data Sheet.

9. For a trial run, activate the Photogate by clicking on the "Start" button; carefully release the ball from the pre-set mark, so that it rolls down the ramp *from rest* and note the approximate point where the ball hits the floor.

10. Using the masking tape, fasten a sheet of the white "Target" paper provided to the floor so that the ball will land near the center of the page at the position marked as "+". Place the Target paper with "Register Line" perpendicular to the path of the ball and closest to the reference mark previously determined using the plumb bob. Measure the distance from the plumb reference mark to the Register Line (L) and record this value on the Data Sheet.

11. Place a sheet of carbon paper faced down over the Target paper so that the point where the ball hits the paper will be marked. The horizontal displacement (R) of the ball is the distance from the plumb reference mark to the Register Line (L) plus the distance from the Register Line to each point of impact (d).

12. Hold the ball at the pre-set point of release near the top of the ramp. "Start" the Photogate and carefully release the ball. Repeat this procedure nine more times to obtain ten readings. Record the corresponding values of the measured velocity of the ball, as recorded by *DataStudio*.

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13. After the completing the data runs, remove the Target paper for inclusion in your Report. Measure the distance *d* from the Register Line to each point of impact. Record these measurements on the Data Sheet.

Lab Report Section

DATA

1. Width of ball through Photogate (x) = __________ cm
2. Mass of the ball, m_b = __________ kg
3. Diameter of the ball = __________ cm
4. Distance along the ramp from the release point to the end of the ramp = __________ cm
5. Angle of inclination of the ramp, θ = __________°
6. Vertical height of the center of the ball above the table at the release point, h_1 = ___________ cm
7. Vertical height of the center of the ball at the end of the ramp, h_2 = ___________ cm
8. Vertical height traveled by the ball from release point to the end of the ramp, $h = h_1 - h_2$ = ___________ cm
9. Vertical height dropped by the ball from the end of the ramp to the plumb Reference mark, Y = ___________ cm
10. Horizontal distance from the plumb reference mark to the Register Line, L = ___________ cm

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11. Data Table:

Trial	Measured velocity, v_m (cm/s)	Deviation (cm/s)	(Impact) distance *d* (cm)	Deviation (cm)
Run #1				
Run #2				
Run #3				
Run #4				
Run #5				
Run #6				
Run #7				
Run #8				
Run #9				
Run #10				
Averages:				

12. Calculated *theoretical* velocity of the ball

    v_t = ______________ cm/s

13. Average measured velocity from Data Table above,

    v_m = ______________ cm/s

14. Calculated average horizontal displacement (range) of the ball,

    R = ______________ cm

15. Calculated ballistic velocity from the average displacement *R*

    v_b = ______________ cm/s

16. Percent error of the average measured velocity relative to the theoretical velocity,

    Percent error = ___________

17. Percent error of the calculated ballistic velocity relative to the theoretical velocity,

    Percent error = ___________

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CALCULATIONS ____

1. Calculate the average value of the measured velocity for the ten data runs of the ball. Calculate the deviation of each measured velocity from the average and then calculate the average deviation (a.d.). Enter your result on the Data Table. Attach the error to the measured velocity as: *average velocity* ± *the a.d.*

2. Compute the average (impact) distance *d* on the Target paper from the Register Line to each impact point and the average deviation (a.d.). Enter this average in the appropriate space in the data table.

3. Compute the vertical distance $h = h_1 - h_2$ through which the center of mass of the ball traveled from the release point to the end of the ramp.

4. Calculate the theoretical velocity of the ball. Enter your answer on the data sheet.

5. Calculate the average horizontal displacement *R* of the ball (from the end of the ramp to the Target paper) by adding the horizontal distance from the plumb reference mark to the Register Line, *L*, to the average value of *d* from the Data Table (as determined in Calculation Step #2 above).

6. Calculate the ballistic velocity of the ball. Enter your answer on the data sheet.

7. Assume the theoretical value of the velocity measured directly to be accurate. Calculate the percent error between the measured velocity and the theoretical velocity. Calculate the percent error between the calculated ballistic velocity and the theoretical velocity.

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QUESTIONS

1. Show the detailed derivation of the equation you used in calculating the ballistic velocity of the ball (radius *r*) after leaving the ramp with velocity v_b at an angle θ below the horizontal, from a vertical height *Y* above the floor, and striking the horizontal floor at a horizontal distance *R*. Draw a diagram! Use a separate sheet.

2. Do your calculated values of the ballistic velocity and theoretical velocity fall within $\pm\ \sigma$ (one standard deviation) of the average value of the velocity measured directly with the Photogate?

3. What factors and reading errors directly influence the velocity values?