The Electrostatic Force

Equipment	Quan
kilovolt power supply	1
digital scale	1
capacitor plates	2
lengths of magnet wire	2
1 $G\Omega$ resistor	1
BB wires & alligator clips	3
Upper plate winching supports	3

When a capacitor is charged, the two plates hold charges of equal magnitude and opposite sign. The magnitude of the charge on each plate is;

$$Q = CV \qquad eq. 1$$

Since the two plates are oppositely charged, they attract each other. We can find an expression for the magnitude of the force. The electrostatic force on a charge in an electric field is given by:

$$F = EQ \qquad eq. 2$$

What's the magnitude of the electric field in the capacitor? Recall that an electric field is the gradient of the potential field. A voltage source creates a potential difference between the plates of a capacitor, and if the field between the plates is homogeneous (in our case it's nearly homogeneous) then the magnitude of the field will be given by:

$$E = \frac{V}{d} \qquad eq. 3$$

Here, V is the potential difference between the plates and d is the distance between them.

We will be concerned with the force on one plate due to the field generated by the other. Equation 3 gives the field due to both plates, and so the field due to one plate is:

$$E = \frac{V}{2d} \qquad eq. 4$$

Finally, the capacitance of a parallel plate capacitor is given by:

$$C = \frac{\epsilon_o A}{d} \qquad eq. 5$$

Using equations $1, 4$ & 5 in equation 2, we have:

$$F = \frac{V}{2d}\frac{\epsilon_o A}{d}V$$

$$F = \frac{\epsilon_o A}{2d^2}V^2 \qquad eq. 6$$

You'll test equation 6 in the experiment. To do this, you'll measure the attractive force F for a number of different voltages V, and then plot F as a function of V^2. Equation 6 predicts that F is proportional to V^2, so the points should lie along a straight line, and the slope of that line should be:

$$slope = \frac{\epsilon_o A}{2d^2} \qquad eq. 7$$

Correction Factor

Equation 6 (the force between the plates) was derived using the assumption that $D \gg d$ (D is the diameter of each capacitor plate). In fact,

$D = 30d$. This ratio is not big enough for the force equation to be accurate to within our ability to measure the force. A correction factor can be determined, but only with some very complicated mathematics. It's given here without proof.

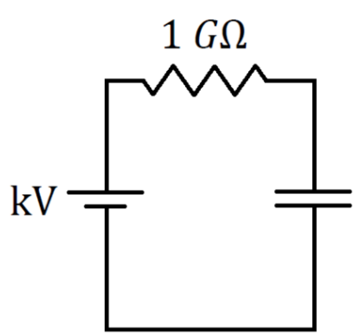
$$F = \frac{\epsilon_o A}{2d^2}\left(1 + \frac{2d}{D}\right)V^2$$

The equation is still not exact. This correction factor is a first order factor, close enough when $D = 30d$. The slope of your graph will then be:

$$slope = \frac{\epsilon_o A}{2d^2}\left(1 + \frac{2d}{D}\right) \qquad eq.8$$

You'll draw the graph, determine the slope, and then, having measured A (the area of one plate) and d (the distance between the plates), you can solve equation 8 for ϵ_o and compare to the known value.

Experimental Circuit



$figure\ 1$

The circuit diagram above illustrates the experimental circuit. kV represents the kilovolt power supply and the capacitor is where the force is measured. The $1\ G\Omega$ resistor protects the power supply from excessive current in case the capacitor plates come into contact.

Setting Up the Apparatus

1. Each of the two Styrofoam capacitor plates should have a piece of magnet wire taped to its surface. If not, this needs to be done.
2. The digital scale won't be turned on until the entire experimental setup is assembled. Remove the screw-off cover from the scale.
3. Place the Styrofoam cylinder on the bed of the scale.
4. Place the lower capacitor plate (the plate with no hooks on it) on the cylinder with the aluminum foil side facing up.
5. Place the upper plate on the lower plate so that the two foil surfaces are touching.
6. Place the 3 winching plate supports around the upper plate so that the suspending hooks are directly above the hooks on the plate.
7. Link the hooks to the plate hooks and winch the upper plate so that it is suspended above the lower plate. The two plates should hang stably such that the suspending threads are vertical and the plates are parallel.
8. Now lift the Styrofoam cylinder and lower plate and press the scale on/off button to turn it on (there must be no weight on the scale when it is turned on). The button should be held in for a second or two.
9. When the scale is on, lower the cylinder and lower plate back onto the scale. It will then read the weight of the two objects.
10. Press the on/off button the zero the reading.
11. Make sure the readout indicates force units (N) instead of mass units.
12. Plug a BB wire into the positive socket of the power supply.

13. Place an alligator clip on the other end of this wire.
14. Clip the 1 $G\Omega$ resistor into the alligator clip and place another clip on its other end, and a BB wire into this clip.
15. A third alligator clip on the second BB wire is clipped to the upper plate wire.
16. Using a BB wire and alligator clip, hook the lower plate wire to the ground socket of the power supply.

Procedure

1. Measure the diameter of one of the capacitor plates and record it in SI units.

$$D = \_\_\_\_\_\_\_\_\_\_ \; m$$

2. From D, calculate the area of the plate.

$$A = \_\_\_\_\_\_\_\_\_\_ \; m^2$$

3. Wire the experimental circuit as in $figure\ 1$ The wires to the capacitor plates will be magnet wire. The whole configuration is shown below.
4. The upper capacitor plate rests on 3 wedge supports. These can be moved toward and away from the plate in order to adjust the plate separation, which you should set as close as possible to 1 cm. Thus:

$$d = 0.010\ m$$

5. Turn on the kV power supply and raise the voltage (V_{ps}) to about $1000\ V$ (it doesn't have to be exactly $1000\ V$ since we won't be using this number in the analysis).
6. Read the voltage from the power supply readout and record it in volts in the table below.
7. Record F, the force indicated on the scale.
8. Follow this procedure for the other five power supply voltages.
9. For each V_{cap}, calculate V_{cap}^2.
10. Plot $F\ vs\ V_{cap}^2$.
11. Determine the slope of the graph.

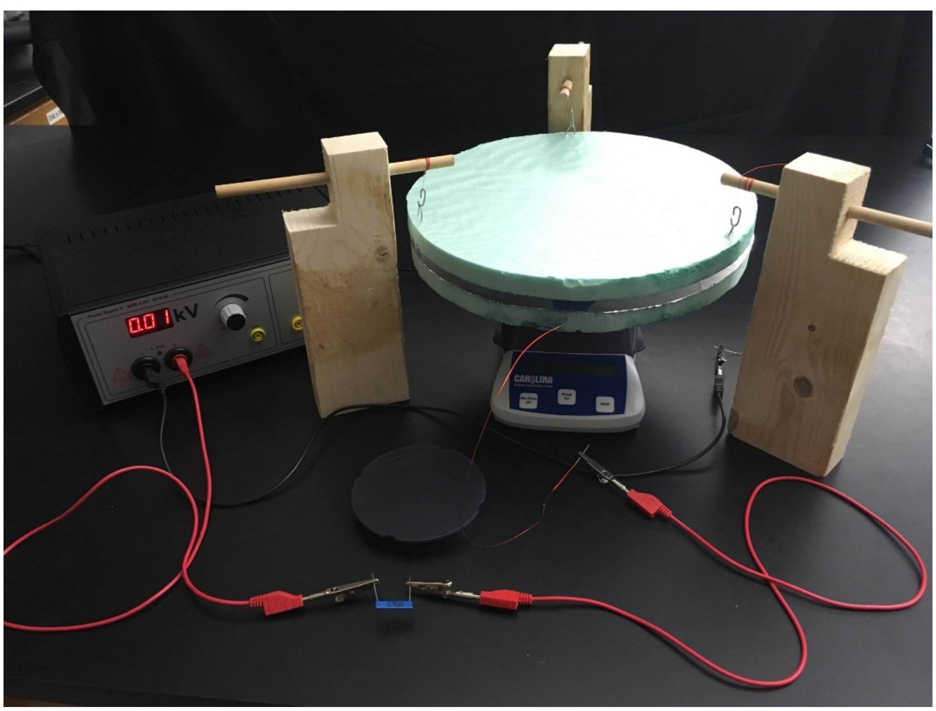
$$slope = \_\_\_\_\_\_\_\_\_\_ \; N/V^2$$

12. Use $equation\ 8$, along with your values for $A, d, \&\ D$, & the $slope$, to calculate ϵ_o.

$$\epsilon_o = \_\_\_\_\_\_\_\_\_\_ \; F/m$$

13. Find the percent error of your value for ϵ_o.

$$\%\ error = \_\_\_\_\_\_\_\_\_\_ \; \%$$



V_{approx}	V $(kV\ readout)$	V^2	F $(dig.\ scale)$
$\approx 1000\ V$			
$\approx 2000\ V$			
$\approx 3000\ V$			
$\approx 4000\ V$			
$\approx 5000\ V$			
$\approx 6000\ V$			
slope			
ϵ_o			
$\%\ error$			