The Effect of Moment of Inertia on Rolling Acceleration

Equipment	Qty.
Marble Ball	1
Wooden Cylinder	1
Stainless Steel Ring	1
Wooden Ramp	1
Wooden Ramp Prop Block	1
Stopwatch	1

When an object rolls down a ramp, it gains kinetic energy in two forms: translational and rotational.

$$KE = KE_t + KE_r$$

$$KE = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

The moments of inertia for a ring, a sphere, and a cylinder are, respectively:

$$I_r = mR^2 \qquad I_s = \tfrac{2}{5}mR^2 \qquad I_c = \tfrac{1}{2}mR^2$$

Exercise 1: Derive the following 3 expressions for the total rolling kinetic energy (translational plus rotational) of a ring, a sphere, and a cylinder. Recall that for a rolling object of radius r, $v = \omega r$. **Do the derivations on a separate sheet of paper, with clear steps. An example of the same derivation for a hollow sphere is at the end of the lab.**

$$KE_r = mv^2$$

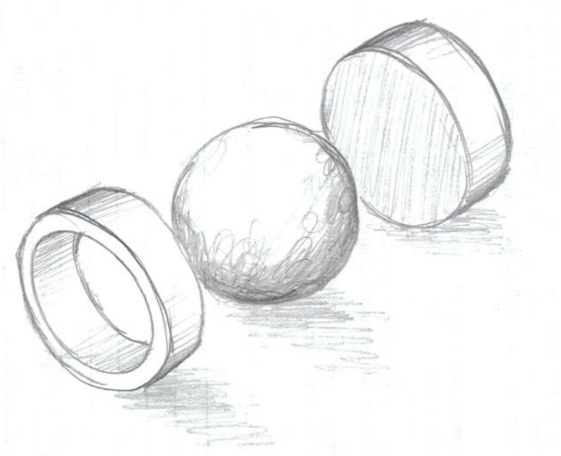
$$KE_s = \frac{7}{10}mv^2$$

$$KE_c = \frac{3}{4}mv^2$$

Note that the ring's rotational kinetic energy is as great as its translational KE, while the sphere's rotational KE is only 2/5 of its translational KE.

Next we'll use conservation of energy to find the speed of each of the 3 rolling objects at the bottom of the ramp. Since it's the gravitational potential energy $PE = mgh$ that provides the final kinetic energy, conservation of energy states that:

$$KE_f = PE_i$$



Here PE_i is the initial potential energy at the top of the ramp and KE_f is the kinetic energy at the bottom.

Exercise 2: Using your kinetic energy equations and conservation of energy, derive the following 3 expressions for the speeds of the 3 rolling objects at the bottom of the ramp. Use h to denote the vertical height of the ramp. **Do the derivations on a separate sheet of paper, with clear steps. An example of the same derivation for a hollow sphere is at the end of the lab.**

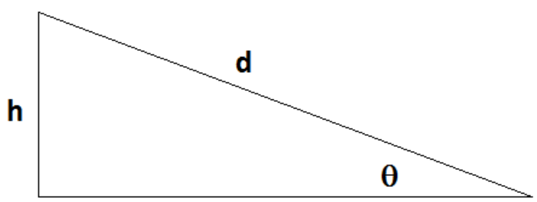
$$v_r = \sqrt{gh}$$

$$v_s = \sqrt{\frac{10}{7}gh}$$

$$v_c = \sqrt{\frac{4}{3}gh}$$

Thus the sphere should get to the bottom fastest, followed by the cylinder, and then the ring.

Next we'll find the time to descend the above ramp for each object. To do this, we'll use one of the equations of kinematics for constant acceleration, namely $d = \frac{1}{2}(v_o + v)t$.



Exercise 3: Use your speed equations and the above kinematic equation to find the following 3 expressions for the times to descend the ramp. Note that *d* is the distance traveled down the ramp, and *h* is the vertical drop. Also note that if the objects start from rest then $v_o = 0$ for each. **Do the derivations on a separate sheet of paper, with clear steps. An example of the same derivation for a hollow sphere is at the end of the lab.**

$$t_r = d\sqrt{4/(gh)}$$

$$t_s = d\sqrt{14/(5gh)}$$

$$t_c = d\sqrt{3/(gh)}$$

Now we can compare the theoretical ramp descent times for the 3 objects. A straightforward comparison between any two of them is their ratio.

Exercise 4: Use your time equations to find the following 3 ratios of the times it takes to descend the ramp. **Do the derivations on a separate sheet of paper, with clear steps. An example of the same derivation for a hollow sphere is at the end of the lab.**

$$\frac{t_r}{t_c} = \sqrt{4/3}$$

$$\frac{t_r}{t_s} = \sqrt{10/7}$$

$$\frac{t_c}{t_s} = \sqrt{15/14}$$

Exercise 5: Time each one for ten trials as it rolls down the ramp, and calculate the average for each.

	Ring	Sphere	Cylinder
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			
Avg.			

Exercise 6: Calculate the ratios of the 3 averages. Then determine the percent errors of each, using the corresponding theoretical ratios as the accepted values in the calculations.

$\frac{t_r}{t_c} =$ _______ % error =_______

$\frac{t_r}{t_s} =$ _______ % error =_______

$\frac{t_c}{t_s} =$ _______ % error =_______

Example Derivations (done for a hollow sphere, which has $I = ⅔mR^2$).

Exercise 1:

$$KE_{hs} = KE_t + KE_r$$

$$KE_{hs} = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$KE_{hs} = \frac{1}{2}mv^2 + \frac{1}{2}(⅔mR^2)\left(\frac{v}{R}\right)^2$$

$$KE_{hs} = \frac{1}{2}mv^2 + ⅓mv^2$$

$$KE_{hs} = ⅚mv^2$$

Exercise 2:

$$KE_f = PE_i$$

$$⅚mv^2 = mgh$$

$$v^2 = \frac{6}{5}gh$$

$$v_{hs} = \sqrt{\frac{6}{5}gh}$$

Exercise 3:

$$d = \frac{1}{2}(v_o + v)t$$

$$d = \frac{1}{2}\sqrt{\frac{6}{5}gh}\, t$$

$$d = \sqrt{\frac{6}{20}gh}\, t$$

$$d = \sqrt{\frac{3}{10}gh}\, t$$

$$t = d\sqrt{\frac{10}{3gh}}$$

Exercise 4:

Find t_r/t_{hs}

$$\frac{t_r}{t_{hs}} = \frac{d\sqrt{4/(gh)}}{d\sqrt{10/3gh}}$$

$$\frac{t_r}{t_{hs}} = \sqrt{\frac{4/(gh)}{10/(3gh)}}$$

$$\frac{t_r}{t_{hs}} = \sqrt{\frac{12}{10}}$$

$$\frac{t_r}{t_{hs}} = \sqrt{\frac{6}{5}}$$