Name ____________________ Class ______________ Date _________

THE SONOMETER

Resonance Modes of a Stretched String

Experimental Objectives:

The objectives of this experiment are to:

(1) Experimentally determine the relationship between the length of a stretched string and the frequencies at which resonance occurs at a constant tension;

(2) Verify the variation of the wave frequency with the tension in the stretched string with a constant length;

(3) Investigate the resonance modes of a stretched string.

Apparatus:

Equipment Needed	Qty	Equipment Needed	Qty
Signal Generator	1	Mass Set (SE-8705)	1
Voltage Sensor (CI-6503)	1	Sonometer with Coils (WA-9757)	1

Theory

A string on a musical instrument, such as a guitar, will vibrate at a specific frequency when plucked. The string vibrates at a different frequency if the length or tension is changed.

Standing Waves

A wave is described by its wavelength λ, the frequency of oscillation *f*, and the wave velocity, *v*. These quantities are related by the fundamental wave equation of wave propagation

$$v = f\lambda$$

If the string is fixed at one end, the wave will be reflected back when it strikes that end. The reflected wave will then interfere with the original wave. Assuming the amplitudes of these waves are small enough so that the elastic limit of the string is not exceeded, the resultant waveform will be just the sum of the two waves:

$$y = 2y_m \sin 2\pi\left(\frac{x}{\lambda}\right)\cos 2\pi\left(\frac{t}{\lambda}\right)$$

This equation has some interesting characteristics. At a <u>fixed time</u>, $\mathbf{t_o}$, the shape of the string is a sine wave with a maximum amplitude of:

$$y = 2y_m \cos 2\pi\left(\frac{t_0}{\lambda}\right)$$

At a <u>fixed position</u> on the string, $\mathbf{x_o}$, the string is undergoing simple harmonic motion, with an amplitude of

$$y = 2y_m \sin 2\pi\left(\frac{x_0}{\lambda}\right)$$

Therefore, at points of the string where $\mathbf{x_o} = \boldsymbol{\lambda/4, 3\lambda/4, 5\lambda/4, 7\lambda/4}$, etc., the amplitude of the oscillations will be a maximum (oscillations from both waves reinforce each other). The points of maximum amplitude are called **antinodes**. At points of the string where $\mathbf{x_o = \lambda/2, \lambda, 3\lambda/2, 2\lambda}$, etc., the amplitude of the oscillations will be zero (oscillations from both waves cancel each other). The points of zero amplitude are called **nodes**.

This waveform is called a **standing wave** because there is no propagation of the waveform along the string. Each point of the string oscillates up and down with its amplitude determined by whether the interfering waves are reinforcing or canceling each other.

Resonance

The analysis above assumes that the standing wave is formed by the superposition of an original wave and one reflected wave. In fact, if the string is fixed at both ends, each wave will be reflected every time it reaches either end of the string. In general, the multiple reflected waves will not all be in phase, and the amplitude of the wave pattern will be small. However, at certain frequencies of oscillation, all the reflected waves are in phase, resulting in a very high amplitude standing wave. These frequencies are called **resonant frequencies**.

In this activity, the relationship between the length of the string and the frequencies at which resonance occurs is investigated. It is shown that the conditions for resonance are more easily understood in terms of the wavelength of the wave pattern, rather than in terms of the frequency. In general, resonance occurs when the wavelength (λ) satisfies the condition:

$$\lambda = 2L/n; \quad n = 1, 2, 3, 4,\ldots$$

Another way of stating this same relationship is to say that the length of the string is equal to an integral number of half wavelengths.

$$(n/2)\lambda = L; \quad n = 1, 2, 3, 4,\ldots$$

This means that the standing wave is such that a node of the wave pattern exists naturally at each fixed end of the string. Only certain discrete wavelengths (which satisfy the equation above) and corresponding to specific modes of vibration are possible for a given length of wire. Possible modes of vibration will be observed for $\lambda = 2L, L, 2L/3, L/2$, etc.

The velocity of a wave on a stretched string is given by

$$v = \sqrt{\frac{T}{\mu}}$$

T is the tension force in the string and μ is the linear mass density (mass per unit length, *m/l)* of the string. From the fundamental equation of wave propagation, $f = v/\lambda$.

Therefore, substituting for the velocity, we obtain the relation that for any given mode of vibration for which $\lambda = 2L/n$,

$$f = \frac{n}{2L}\sqrt{\frac{T}{\mu}}.$$

This experiment will be conducted using a *Sonometer* to investigate how the frequencies of vibrating wires vary with length, density, and tension. The stretched *Sonometer* wire (under tension) is set into vibration and “tuned” by means of “driver coil” connected to a signal generator. The vibrating wire can be “tuned” to the “pitch” of a known frequency by varying either the vibrating length (L) or the tension (T) or both. The resonant vibrations of the wire are detected with a "detector coil" placed under the vibrating wire. The length of the vibrating wire may be varied by the use of two metal bridges, inserted under the wire at any desired point, to set up nodes at these bridge points. The output signal of this detector coil is displayed in the *DataStudio* program.

PROCEDURE:

A: Sonometer and Computer Setup

In this activity, use a Driver Coil connected to the Power Amplifier to vibrate a thin wire that is stretched over two "bridges" on a Sonometer. Use the Signal Generator in *DataStudio* controls the frequency at which the wire is vibrated. Use a Voltage Sensor connected to the Detector Coil on the Sonometer to measure the amplitude of the vibrating wire.

Use the *DataStudio* program to display the output signal that controls the Driver Coil, and the input signal from the Detector Coil. Determine the resonant frequencies of the stretched wire by watching the amplitude of the input signal from the Detector Coil.

SAFETY REMINDER	
• Follow all safety instructions. Be very careful while hanging the 2-kg mass on the tensioning lever. The wire could break if the tension is too high. Do not DROP the weight hanger on the tensioning lever.	THINK SAFETY ACT SAFELY BE SAFE!

1. Connect the *ScienceWorkshop* interface to the computer, switch on the interface, and switch on the computer.
2. Connect one Voltage Sensor DIN plug to Analog Channel A of the interface.
3. Connect the second Voltage Sensor DIN plug to Analog Channel B of the interface and the banana plugs to the output of the Signal Generator.

4. Open the document titled as shown:

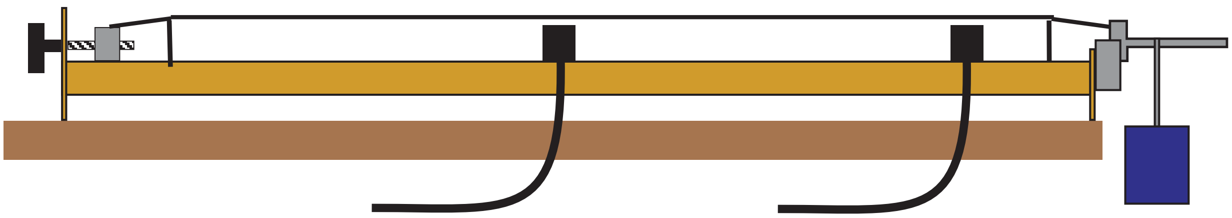
P43 Sonometer.DS

- The *DataStudio* document also has a Workbook display. Read the instructions in the Workbook.
- The Scope display is set to show the voltage from the Signal Generator (Voltage, ChB) and the voltage from the detector coil (Voltage, ChA).

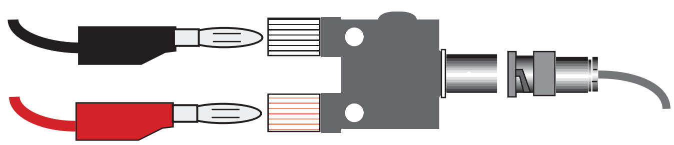
B: Sensor Calibration and Equipment Setup

- Neither the Voltage Sensors nor Signal Generator need to be calibrated.

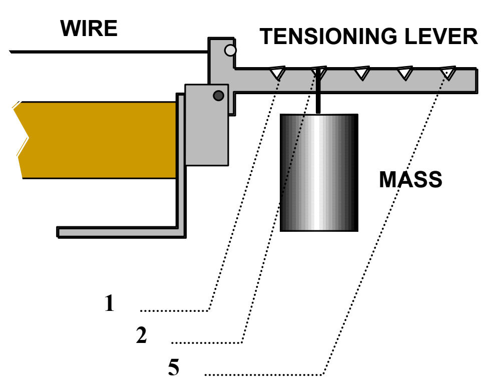
1. Set the Sonometer at the edge of a table so the tensioning lever extends beyond the table.
2. Start with the bridges **60** cm apart. Select one of the wires that is included with the Sonometer. Attach the wire to the peg on the cylinder with the string adjustment knob, and



   to the rounded slot in the vertical section of the tensioning lever.
3. Position the Driver Coil approximately 5 cm from one of the bridges. Connect the Driver Coil banana plugs into the Signal Generator output jacks.
4. Position the Detector Coil near the center of the wire between the two bridges. Attach the BNC connector on the Detector Coil cable to the BNC-to-banana-jack Adapter Plug.



   Connect the Voltage Sensor banana plugs into the jacks on the Adapter Plug.
5. Hang a mass of approximately 1 kg from the <u>SECOND</u> notch (2) in the tensioning lever. The notches on the lever are numbered from the end of the wire outward, as shown, to the FIFTH notch. Use the string adjustment knob to tighten or loosen the wire until the tensioning lever is <u>horizontal</u>.



6. Calculate the tension in the wire by multiplying the number of the notch on the tensioning lever by the weight of the hanging mass [mass (kg) x 9.8 N/kg], (i.e. tension = *notch#* x *mass* x '*g*')
7. Measure the diameter of the wires using a micrometer caliper.
8. Record the diameter of the wire, tension, and linear density of the wire in the Data Sheet section.

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C: Data Acquisition

1. Fundamental Frequency (Natural mode)

- The fundamental frequency is the natural frequency at which the wire vibrates when it is plucked.

(a) Start with the vibrating length of 60 cm between the bridges as in the previous activity. The hanging mass of approximately 1 kg should be at the SECOND notch in the tensioning lever.

(b) Position the Driver Coil approximately 5 cm from one of the bridges near the tensioning lever. Position the Detector Coil near the center of the wire between the two bridges.

(c) Use the Frequency Spectrum (FFT) display to measure the approximate value of the fundamental frequency of the wire on the Sonometer.

(d) Start measuring. 'Start' in DataStudio to begin monitoring data.

(e) Pluck the wire near the center of its vibrating length. As the wire vibrates, the Frequency Spectrum (FFT) display will show the fundamental frequency recorded by the detector coil.

(f) Click 'STOP' to end the measuring and "freeze" the frequency display in time. Use the 'Smart Tool' (DataStudio) to find the fundamental frequency of the wire.

(g) Record this fundamental frequency of the wire.

2. Fundamental Frequency (Lowest Resonant frequency)

- The fundamental frequency will be the lowest resonant frequency at which the wire vibrates at maximum amplitude due to the magnetic force from the driver coil.
- Frequencies that result in maximum amplitude (string vibration) are resonant frequencies. When resonance occurs, the voltage from the Detector Coil will be at its maximum amplitude.

(a) Set the vibrating length equal to 60 cm between the bridges, as in the previous activity. The hanging mass of approximately 1 kg should be at the SECOND notch in the tensioning lever.

(b) Position the Driver Coil approximately 5 cm from one of the bridges near the tensioning lever, but within the vibrating section of the wire. Position the Detector Coil near the center of the wire between the two bridges.

(c) Set up the Signal Generator. Select the appropriate frequency range on the Signal Generator. Using the "FREQ" or "ADJUST" knob to set the frequency to a value that is approximately one-half of the fundamental frequency of the wire. (For example, if the approximate fundamental frequency is 110 Hz, set the frequency in the Signal Generator window to 55 Hz.).

- NOTE: The reason that the driving frequency in the Signal Generator should be approximately one-half of the fundamental frequency is because the driver coil (an electromagnet) pulls on the metal wire TWICE per cycle. *(Why?)* Therefore, if you set the driving frequency at 60 Hz, the wire will vibrate at 120 Hz.

(d) Click 'Start' to begin measuring again. Observe the middle area of the wire, and the traces on the Scope display.

(e) Slightly adjust the signal frequency up and down. Watch the wire and the traces on the Scope. The lowest frequency at which resonance (and maximum amplitude) occurs is the first, or fundamental, resonant mode.

(f) Record this lowest resonant frequency on the Signal Generator obtained for the Fundamental Resonant Mode.

3. Variation of Resonant Frequency with the vibrating Length of the wire

(a) Repeat ***Procedure C.2*** with vibrating lengths of 50 cm, 40 cm, and 30 cm by moving the bridges appropriately.

(b) For each setting of the length, position the Driver Coil approximately 5 cm from one of the bridges near the tensioning lever and the Detector Coil near the center of the wire between the two bridges.

(c) Record the resonant frequencies with the corresponding vibrating lengths.

4. Variation of Resonant Frequency with the Tension on the wire

(a) As performed in ***Procedure C.2***, reset the vibrating length to 50 cm with the hanging mass still in the second notch. Keep this vibrating length constant for this procedure.

(b) Change the tension by moving the mass to the first notch on the lever. Use the string adjustment knob to tighten or loosen the wire until the tensioning lever is horizontal.

(c) Search for the lowest resonant frequency as performed in ***Procedure C.2***.

(d) Repeat these steps with the mass hanging in the third, fourth, and fifth notch. Record the matching lowest resonant frequencies.

Record all your data in the Data Sheet section

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DATA SHEET: The Sonometer

1.

Diameter of wire = ______________ m Tension (at 2nd notch) = _________ N

2. *Mass per unit length (using equation provided), μ_1 = ____________ g/m

* The linear density of the wire, μ, is given by: $\mu = [(\text{diameter in cm})^2 \times 604.52]$ g/m

<u>Data for Procedure #C.1 and C.2</u>:

3. Fundamental Frequency of the wire (natural mode): _____________ Hz.

4. Fundamental Frequency of the wire (lowest resonant mode): _____________ Hz.

<u>Data Table for Procedure #C.3</u>:

5. Constant Wire Tension (at 2nd notch) = ___________ N

6.

Signal Generator frequency	Resonant frequency of Wire, (*f*)	Vibrating length of the wire (*L*)	Reciprocal of length of wire (1/*L*)

7. Mass per unit length (from graph of frequency versus length), μ_2 = _____________ g/m

8. Percent error (compared with μ_1 as theoretical value) = ________________

<u>Data Table for Procedure #C.4:</u>

9. Vibrating length of the wire (*L*): __________ cm

Notch #	Tension (*T*)	Signal Generator frequency	Resonant frequency of Wire, (*f*)	Square of Frequency, $(f)^2$
1				
2				
3				
4				
5				

10. Mass per unit length (from graph of frequency^2 versus tension), μ_3 = ____________ g/m

11. Percent error (compared with μ_1 as theoretical value) = _______________

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<u>Calculations:</u> The Sonometer

1. Calculate the linear mass density of the wire (mass per unit length) from the given formula and enter the result in the data sheet as μ_1.

2. From the data of Procedure #3, calculate the reciprocals of the length of the wire. Plot a graph using these reciprocals as abscissas and the corresponding frequencies as ordinates. Determine the slope of this graph. From the value of this slope, calculate the linear mass density of the wire, μ_2. Calculate the percent error of the linear density as compared to μ_1.

3. From the data of Procedure #4, calculate the squares of the frequency readings. Plot a graph using the values of the tension as abscissas and the corresponding squares of the frequency as ordinates. Determine the slope of this graph. From the value of this slope, calculate the linear mass density of the wire, μ_3. Calculate the percent error of the linear density as compared to μ_1.

4. From the data of Procedures #5 and #6, calculate the wavelengths for the harmonics and enter these values in the data table.

<u>Post-Lab Questions</u> – The Sonometer

1. What is relationship between the length of a stretched string and the frequencies at which resonance occurs?

2. On what physical properties do the following musical terms depend: (a) the pitch, (b) the loudness, and (c) the quality or timbre.

3. What is the difference between an "*overtone*" and a "*harmonic*"?

4. A sonometer wire of linear mass density $\mu = 0.30$ g/m vibrates with a fundamental frequency of 200 Hz. If the vibrating length of this wire is 50 cm, calculate the tension on the wire. What mass hung on the wire would produce this tension?

5. What is the relationship between the number of antinode segments and the number of the resonant mode?

6. The first overtone of a stretched wire has a frequency of 200 Hz when the wire is stretched by a mass of 5 kg. If the vibrating length of the wire is 1.0 m, find the linear mass density (kg/m) of the vibrating wire.

