Name _____________________ Class ______________ Date _________

Equilibrium of a Rigid Body
(Torques and Rotational equilibrium)

Overview

When a system of forces, which are not *concurrent*, acts on a *rigid* object, these forces will tend to move the object from one position to another (*translation*) and may also produce a turning effect of the object around a given axis (*rotation*). Thus, the action of these non-concurrent forces on the rigid object may result in either a linear acceleration during translation or angular acceleration during rotation. Translation of the object along any axis is produced when there is an unbalanced or excess force (F_{net}) along that axis. This non-zero net force produces a linear acceleration, $a = F_{net}/m$, where m is the mass of the object. The turning effect caused by an applied force about a given axis is defined as the *torque* or *moment* of the force. The magnitude of this torque (a vector) is equal to the product of the magnitude of the force times its lever arm (the perpendicular distance from the axis of rotation to the line of action of the force).

$$\sum \vec{\tau} = \vec{r}_1 \mathrm{x} \vec{F}_1 + \vec{r}_2 \mathrm{x} \vec{F}_2 + .. = 0$$

Clockwise torques (usually assigned a negative sign) are those that tend to rotate the object in the clockwise sense while the *Counterclockwise* torques (usually assigned a positive sign) tend to rotate the object in the counterclockwise sense about the axis of rotation. Rotational motion of the object results when there is a non-zero net torque (τ_{net}) about an axis of rotation. This non-zero net torque produces an angular acceleration, $\alpha = \tau_{net}/I$, where I is the moment of inertia of the object. Therefore, a rigid object is in equilibrium when it has a zero linear acceleration and also a zero angular acceleration!

In this laboratory experiment, you will investigate the condition that is necessary to maintain static equilibrium of a rigid object. The meter stick (though not completely rigid) will be considered as the rigid object, which will be subjected to a system of *coplanar*, non-concurrent forces. If we assume that the mass this meter stick is uniformly and symmetrically distributed along its length, then we can define the *center of gravity* of the meter stick to be located at its (geometric) center of symmetry. The sum of all the torques due to the differential elements of the mass of the stick will be zero about an axis of rotation through this center of gravity. In a uniform gravitational field, when the acceleration due to the force of gravity (g) is constant, the center of gravity of any symmetric object of uniform mass density coincides with the center of mass of that object. From the measurements of the positions and magnitudes of the forces on the meter stick, you should be able to apply the conditions for equilibrium in order to:

(a) Locate the center of gravity of the meter stick;
(b) Find the mass of the meter stick by applying known torques on the meter stick;
(c) Compare the torques due to known forces acting on the meter stick;
(d) Determine the mass of an unknown object by method of torques.

Name ____________________ Class _____________ Date _________

SAFETY REMINDER:
- **Follow directions for using equipment.**
- **Your footwear must cover your toes to minimize injury just in case a weight accidentally drops on your foot.**

THINK SAFETY
ACT SAFELY
BE SAFE!

Theory

Two conditions for the mechanical equilibrium of a rigid body acted upon by a system of coplanar parallel forces are stated as follows:

(1) **The vector sum of all the forces acting on the object must be zero.** Mathematically, $\sum \vec{F} = 0$. This condition ensures that the object remains at a particular location and does not moving linearly (unless it is already in motion with a constant uniform velocity).

(2) **The algebraic sum of the torques about any axis of the object must be zero.** Mathematically, $\sum \vec{\tau} = 0$. This condition requires that the sum of the counterclockwise torques must be equal to the sum of the clockwise torques about the axis of rotation. Therefore when the net torque is zero, the object remains in rotational equilibrium. This implies that either the object does not rotate (rotational static equilibrium) or that it rotates with a constant angular velocity (dynamic equilibrium).

In considering the meter stick as a rigid object, we assume that, as long as the stick is unbroken, the relative distances between parts of the stick remain fixed and do not change no matter what the external forces that may be applied on the stick. Therefore, the rigid object will transmit an applied force undiminished throughout its mass and its internal forces are not considered.

Consider the meter stick for this experiment supported on a knife-edge clamp at position *B* on the diagram below. The weight W_s of the meter stick (considered as a uniform beam) may be considered as acting at a point C (the center of gravity of the meter stick at a distance x) from the support. Two other masses hung from the clamps on the stick exert forces equal to the forces of gravity (weights) of the masses, $W = mg$, $W_1 = m_1 g$, *etc*. The support also exerts an upward force F_s at the point of support (*B*). The typical setup of the experiment is shown in the diagram below. If the arrangement shown is assumed to be in

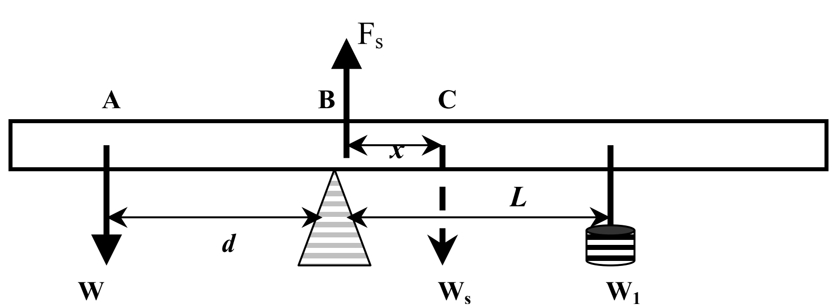

Fig. 1

equilibrium, then from the first condition for equilibrium, the upward force F_s at the support must be equal to the sum of the downward forces.

$$F_{net} = F_s - mg - M_s g - m_1 g = 0$$

From the second condition, the net torque about an axis perpendicular to the plane through the point of support (B) must be equal to zero.

$$\tau_{net} = mgd - M_s gx - m_1 gL = 0$$

Apparatus:

Meter stick	Triple-beam Balance
Three meter-stick knife edge clamps	Set of weights and hangers
Support stand for meter stick	Unknown mass

PROCEDURE:

Part 1: General Experimental Setup

1. Weigh the weight hanger and record its mass. Weigh the three meter-stick clamps (two clamps should have clips) and record the mass of each clamp. The clamps may have slightly different masses. Record the mass of each clamp separately.

2. Determine the mass of the meter stick (M_s) using the triple-beam balance provided and record this value to an accuracy of 0.1 gram.

3. Locate the center of gravity of the meter stick. Slide the supporting (knife-edge) clamp on the meter-stick and balance the meter stick alone on the support stand. Adjust the position of the clamp by sliding the meter stick until the stick is balanced horizontally with no weights attached. Tighten the clamp screw at the balance position. (The head of the tightening screw on the clamp should point downward. Why?). Record this knife-edge clamp position to on the meter stick. This is the center of gravity of the meter stick when measured from the "zero-cm" end of the stick.

Name ____________________ Class ______________ Date _________

Data for Part 1:

Items	Mass of Weight Hanger (g)	Mass of clamp (g)	Total mass (g)
#1			
#2			
#3 (support clamp)			

Measured Mass of meter stick, M_s = _______________ g

Position of the support clamp on meter stick = ___________ cm

Part 2: Finding the Mass of the Meter Stick by Method of Torques

4. Place a clamp at the 10-cm mark and suspend a 200-g mass from the clamp. Loosen the knife-edge clamp and slide the meter stick through the clamp in order to balance the meter stick. Tighten the clamp at the best balance position. Record the new position of the knife-edge clamp (to the nearest 0.5 mm) at which the meter stick is supported.

Data for Part 2:

Mass suspended at the 10-cm mark = ___200 g__
Position of the supporting clamp on the meter stick = ___________ cm

Part 3: Comparison of Torques due to Two Known Forces

5. Place a clamp at the 10-cm mark and suspend a 200-g mass on it. Loosen and slide the knife-edge clamp on the meter stick to support the meter stick at the 40-cm mark. Tighten the supporting clamp at the new position. Place another clamp loosely near the 75-cm mark and suspend a 100-g mass on this clamp. Slide the clamp (with the 100-g mass attached) along the meter stick and experimentally determine (to the nearest 0.5 mm) the position at which this 100-g mass must be placed to balance the system. Record this position of the 100-g mass.

Data for Part 3:

Position of the supporting clamp on the meter stick = _*40-cm mark*
Position of the 100-g mass on the meter stick = __________*cm*

Name _____________________ Class ______________ Date _________

Part 4: Determination of Unknown Mass by Method of Torques

6. Readjust the support clamp to the center of gravity of the meter stick. Suspend an unknown mass from a clamp at the 10-cm mark. On the other side of the meter stick, suspend a 200-g mass from a clamp and adjust the position of this mass until a position of equilibrium is found. Record this position.

7. Remove the unknown mass and weigh it using a laboratory balance. Record the value.

Data for Part 4:

Position of the Unknown mass on the meter stick = __10-cm mark
Position of the 200-g mass on the meter stick = __________*cm*
Mass of unknown object (using laboratory balance) = __________g__

Name ____________________ Class _____________ Date _________

Lab Report Section

Calculations & Results ________________________

Part 1: General Experimental Setup

1. Measured Mass of meter stick, M_s = ______________ g

2. Location of the center of gravity of the meter stick = __________ cm

Part 2: Finding the Mass of the Meter Stick by Method of Torques

3. Position of the support clamp on meter stick = __________ cm

Total Mass at the 10-cm mark (g)	Moment arm for the force at the 10-cm mark (cm)	Moment arm for the force of gravity on the meter stick (cm)

4. By carefully considering the clockwise and counterclockwise torques due to all the applied forces on the meter stick about the point of support, calculate and record the mass of the meter stick. Show your calculation below.

   Calculated Mass of meter stick, M_{exp} = ______________ g

5. Calculate and record the percentage difference between this calculated value and the measured value of the mass of the meter stick obtained using the laboratory balance.

   Percentage Difference between the measured and calculated values of the meter-stick mass = ____________ %

Name ____________________ Class _____________ Date _________

Part 3: Comparison of Torques due to Two Known Forces

6. Calculate the moment arm for all the forces about the point of support. Recall that since the support is not at the center of gravity, the meter stick does not have a zero moment arm and will contribute to the torques. Record the values of the moment arm of the forces.

   Position of the support clamp on meter stick = 40 cm

   Balance position of the 100-g mass on the meter stick = ___________ cm

   Center of gravity of the meter stick (from Calculation 2 above) = __________ cm

Total mass at the position of the 200-g mass (g)	Moment arm of the force at the position of the 200-g mass (cm)	Total mass at the position of the 100-g mass (g)	Moment arm of the force at the position of the 100-g mass (cm)	Moment arm for the force of gravity on the meter stick (cm)

7. Calculate and record the sum of the counterclockwise torques ($\sum \tau_{ccw}$) about the point of support. Use a value of the acceleration due to gravity, $g = 9.8$ m/s^2. Calculate and record the sum of the clockwise torques ($\sum \tau_{cw}$). Calculate and record the percent difference between the torques. Show your calculations below.

   Sum of the counterclockwise torques, $\sum \tau_{ccw}$ = ______________ Nm

   Sum of the clockwise torques, $\sum \tau_{cw}$ = ______________ Nm

   Percent Difference = ___________ %

8. Based on the equilibrium condition when the system is balanced for this Part 3, apply the First Condition for Equilibrium to calculate the upward force F_s exerted by the stand at the point of support. Record this value of the support force, F_s. (Do not ignore the support clamp).

   Force directed upward at the support stand, F_s = ___________ N

Name ____________________ Class ______________ Date _________

9. According to theory, once equilibrium is established, the net torque will also be zero about an axis of rotation perpendicular to the plane through a point at the "0-cm" end of the meter stick. Therefore, consider the torques <u>about the left (0-cm) end of the meter stick</u> and follow Calculation 6 to find new values of the moment arm of the forces. Following Calculation 7, calculate and record the percent difference between the sums of the counterclockwise and clockwise torques. *Draw a neat sketch of the experimental setup and show your calculations below.

   Sum of the counterclockwise torques, $\sum \tau_{ccw}$ = _______________ Nm

   Sum of the clockwise torques, $\sum \tau_{cw}$ = _______________ Nm

   Percent Difference = ____________ %

Name ______________________ Class _______________ Date _________

Part 4: Determination of Unknown Mass by Method of Torques

10. Compute the value of the unknown mass by setting the sum of the counterclockwise torques = the sum of the clockwise torques. Mathematically, $\sum \tau_{ccw} = \sum \tau_{cw}$. Show your calculations below.

Center of gravity of the meter stick = __________cm

Calculated value of Unknown mass = ____________ (g)

Measured value of Unknown mass = ____________ (g)

Percent Difference = ________________ %

Name _____________________ Class ______________ Date _________

Post-Lab Questions

1. Under what conditions can an object considered as a "*rigid body*"?
2. State the two conditions for the mechanical equilibrium of a rigid body acted upon by a system of coplanar parallel forces.
3. In Calculation 7 and Calculation 9 of this experiment, consider the percent difference between $\sum \tau_{ccw}$ and $\sum \tau_{cw}$. Are these experimental results sufficiently good to consider that the data verify the second condition for the equilibrium of a rigid body ($\sum \vec{\tau} = 0$)? Explain.
4. Explain (using Calculation 9) how this experiment demonstrates the validity of the first condition for the equilibrium of a rigid body ($\sum \vec{F} = 0$).
5. What was the advantage of attaching the support clamp on the meter stick at its center of gravity in Procedure-Part 4 in order to find the unknown mass?
6. In the calculations related to Part 1 and Part 2 of the Procedure, the mass of the knife-edge clamp at the point of support and the support force F_s exerted on the meter stick by the stand were apparently ignored. Is this an approximation, or is there some reason why these values do not contribute to the torque? Discuss.