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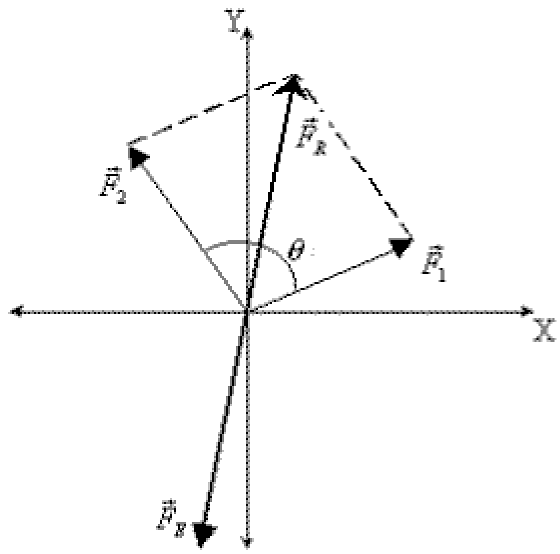
Addition and Resolution of Vectors
Equilibrium of a Particle

Overview

When a set of forces act on an object in such a way that the lines of action of the forces pass through a common point, the forces are described as *concurrent forces*. When these forces lie in the same geometric plane, the forces are also described as *coplanar forces*. A single equivalent force known as the *resultant force* $\vec{F}_R$ may replace a set of concurrent forces $\vec{F}_1$ and $\vec{F}_2$, as shown. This resultant force is obtained by a process of vector addition of the original force vectors and produces the same effect as the combined effect produced by all the original forces. Conversely, a set of concurrent forces can be balanced exactly by a single force that acts at the common point of concurrence of the forces. Such a force is known as the *equilibrant* $\vec{F}_E$ of that set of forces and it is equal in magnitude but acts in exactly opposite direction to the resultant of the set of forces. A particle is considered to be in (static) *equilibrium* under the action of a set of forces when the vector sum of all the forces is zero.



In this laboratory experiment, the student will be introduced to methods of addition of vectors. Using a *force table*, the student will determine the magnitudes and directions of applied concurrent forces, find the resultant force of a given set of vectors, investigate the relationship between the resultant force and the equilibrant force of a given set of forces, and compute the rectangular *x- and y-components* of known forces and their resultant force.

The *force table* apparatus used in this experiment has a horizontally mounted circular table and the rim of this table is calibrated in degrees, from 0° to 360°. Forces of any chosen magnitude can be applied to a central ring (placed around a central pin at the center of the circular table) at any preferred angle by means of strings passing over a pulley and attached to a weight hanger. Each pulley can be adjusted to any chosen position around the rim of the circular table. The force exerted on the central ring is due to the gravitational force acting on the total mass on the weight hanger (including the hanger mass). Adding or removing masses on the weight hanger changes the magnitude of a force vector while moving the position of the pulley changes the direction of the force vector.

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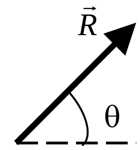
Theory

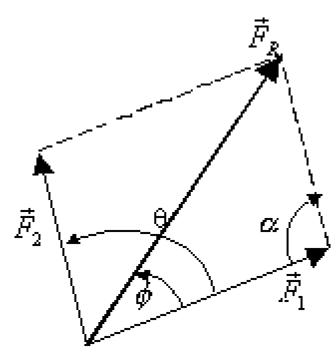
Certain physical quantities, known as *vectors*, can only be completely described in terms of their magnitude and direction. Those quantities that can be completely described only in terms of their magnitudes are called *scalars*. A vector R is represented symbolically as $\vec{R}$ and graphically as an arrow (drawn to scale on graph paper). The length of the arrow is proportional to the magnitude of the vector (R) while the tip of the arrow points in the direction of the vector. The direction may be specified as at angle θ relative to the 0° reference.



When adding only two concurrent force vectors, the resultant force may be determined by the (graphical) “parallelogram method” or by the (analytical) method of components. For two concurrent forces ($\vec{F}_1$ and $\vec{F}_2$) acting on the center ring (considered as a particle) on the force table, the resultant force, $\vec{F}_R = \vec{F}_1 + \vec{F}_2$.

To add these force vectors graphically, a parallelogram (drawn to scale) is constructed with the forces ($\vec{F}_1$ and $\vec{F}_2$) as the adjacent sides with a common point (origin) where the “tails” of the vectors meet. The arrow diagonal of the parallelogram is the resultant force and its magnitude (length) and direction can be measured directly (as the angle ϕ between $\vec{F}_1$, and the resultant force, $\vec{F}_R$) from the vector diagram with a ruler and a protractor.

The magnitude and direction of the resultant force may also be determined analytically by using the *law of cosines* and the *law of sines*. Knowing the angle θ between the forces and with the given magnitudes of $\vec{F}_1$ and $\vec{F}_2$, the appropriate form for the law of cosines for the magnitude of the resultant force, $\vec{F}_R$ is



$$F_R{}^2 = F_1{}^2 + F_2{}^2 + 2F_1F_2\cos\theta$$

or from the parallelogram shown,

$F_R{}^2 = F_1{}^2 + F_2{}^2 - 2F_1F_2\cos\alpha$. It should be left as an exercise for the student to prove that since both formulae are correct, then $\cos\theta = -\cos\alpha$.

Knowing the angle ϕ (shown in the diagram) between force $\vec{F}_1$ and the resultant force, $\vec{F}_R$, the *law of sines* can be applied to determine the direction of the resultant in this case as

$$\frac{F_R}{\sin\alpha} = \frac{F_2}{\sin\phi}.$$

Vector Resolution (Component Method): For a system of concurrent forces acting on a particle, the origin of a rectangular coordinate system can be set up at the common point of concurrence. Therefore, for all the concurrent forces, the component of each force can be resolved along the *x*- and *y*-axes by means of the *sine* and *cosine* functions. The sum of all the

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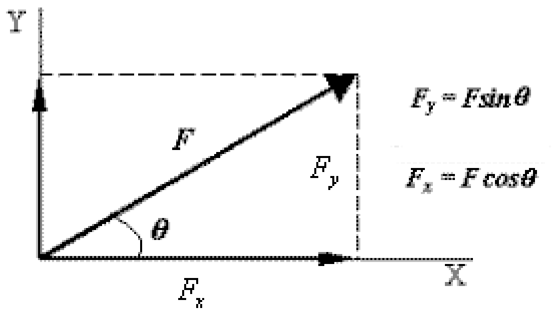
components along the x-axis (F_x) will become the *x-component* of the resultant vector while the sum of all the components along the y-axis (F_y) becomes the *y-component* of the resultant. The magnitude of the resultant can be found using Pythagorean theorem. The direction of the resultant is calculated from the arc tangent (tan^{-1}) of the components.



$$F = \sqrt{F_x^{\,2} + F_y^{\,2}}$$

$$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right)$$

Apparatus:

Force Table (complete with centering pin, ring, pulleys, and strings)	
Construction level or inclinometer	**Set of weights and hangers**
Graph paper	**Protractor and ruler**

SAFETY REMINDER • **Your footwear must cover your toes to minimize injury just in case a weight accidentally drops on your foot.** • **Follow directions for using equipment.**	**THINK SAFETY** **ACT SAFELY** **BE SAFE!**

PROCEDURE:

Methods of Addition of Vectors

1. Set up the force table with the centering pin and the ring in place.

   (a) Use the construction level or the inclinometer to ensure that the force table is level. Make any necessary adjustments using the leveling screws on the supporting tripod base of the force table.

   (b) Use large loose loops to attach one end of each string to the ring placed around the centering pin on the force table. Loose attachment loops will allow the strings to slip freely around the ring. Under the action of the applied forces, the ring should be able to pull away from the center. The other ends of the strings, which run over the pulleys at the edge of the force table, are attached to suspended weight hangers.

(c) When the force system is balanced and therefore in equilibrium, the ring should remain at the center of the force table without the presence of the centering pin. All the strings should be directed exactly at the center of the ring.

1. Graphical Addition of Vectors: Parallelogram Method

2. The student will determine the magnitude and direction of the resultant force (F_R) graphically, using the parallelogram method.

(a) Clamp a pulley at the 30°-mark on the force table and place a 100-g mass on the weight hanger suspended at that position. Calculate the force, in units of newtons (N), produced by the total mass suspended from the weight hanger. Remember to include the mass of the weight hanger in the total mass. Assume three significant figures for all force calculations. Record the calculated force as F_1 in Table #1.

(b) Clamp a second pulley at the 120°-mark on the force table and place a 200-g mass on the weight hanger. Calculate and record the force as F_2 in Table #1.

(c) Using a ruler, carefully construct a diagram of these forces on the force table on *polar coordinates*, with the common point of concurrence being at the origin (the center of the ring). Use a scale of 1 cm = 0.2 N. The length of each vector must be drawn radiating outward from the center and proportional to the vector being represented. The angles between the force vectors must be equal to the angles between the strings on the force table. Label this diagram as *Force Diagram* 1 and include it in your lab report.

(d) On rectangular coordinates, draw a scaled vector diagram of the forces, F_1 and F_2. Label this diagram as *Force Diagram* 2. Use a scale of 1 cm = 0.2 N. The finished vector diagram should fill about half a sheet of the graph paper. Using a ruler and a protractor, measure the magnitude and direction of the resultant force and record the results in Table #1.

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3. The student will determine the magnitude of the equilibrant force, (F_E) experimentally.

   (a) On the force table, clamp a third pulley at 180° from the measured direction of the resultant force.

   (b) From the masses on the suspended weight hanger, determine the magnitude and direction of the equilibrant force needed to keep the center ring in equilibrium.

   (c) Record these values in Table #1.

Table #1:

Mass (kg)	Force	Magnitude (N)	Direction	Length of vector (cm)
0.100	F_1			
0.200	F_2			
	Resultant, F_R			
	Equilibrant, F_E			

2. Addition of Vectors: Analytical Method

4. From the parallelogram of force vectors, F_1 and F_2, constructed in Procedure #2 above, calculate the magnitude of the resultant force using the *Law of Cosines*. Determine the direction of the resultant force (ϕ) relative to a given force vector (F_1) using the *Law of Sines*. Calculate the magnitude of the resultant (F_R) and determine its direction measured relative to the 0° reference mark on the force table. Record these values in Table #2.

Table #2:

F_1 =	
F_2 =	
θ (Angle between F_1 and F_2)	
Calculated Resultant, F_R =	
Direction of the Resultant ϕ =	

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3. Resolution of Vectors: Method of Components

5. The student will resolve a given force vector (*F*) due to 200-g mass at 60° into its (rectangular) *x*- and *y*-components experimentally and analytically.

   (a) Clamp three pulleys at 240°, 90°, and 0° on the force table. Place a total mass (including the hanger) of 200 g on the 240° pulley string. Add masses to the weight hangers at the 90° and 0° positions until the system is in equilibrium. Record the masses in Table #3. Calculate the magnitudes of the corresponding forces.

   (b) Draw a vector diagram to scale, on rectangular coordinates, showing the calculated components of the force (F_x) at 0° and (F_y) at 90°, and the equilibrant force (F_E) at 240°. Label this diagram as *Force Diagram* 3. Record the magnitude and direction of the resultant force in Table #3. Using trigonometry, calculate the components of the given force (*F*) analytically and record them in Table #3.

Table #3:

Force	Mass (kg)	Experimental Magnitude of force (N)	Direction of force	Analytical Components of force (N)
F_x			0°	
F_y			90°	
Equilibrant, F_E	0.200	X	240°	X
Resultant, F	X			

6. The student will calculate the resultant of three forces, theoretically, by method of components, and compare the magnitude and direction of this resultant force with the actual experimental values from the force table.

   (a) Clamp a pulley at 30° with a suspended 100-g mass (label as F_3) and clamp another pulley at 120° with a 200-g mass (label as F_4) similar to the setup in Procedure #4. Clamp a third pulley at 220° with a 150-g mass (label as F_5) on it.

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(b) Calculate the forces produced by these masses and record them in Table #4 as F_3, F_4, and F_5.

(c) Draw a scaled vector diagram of these forces on rectangular coordinates. Label this diagram as *Force Diagram* 4. Use a scale of 1 cm = 0.2 N.

(d) Graphically determine the magnitude and direction of the resultant force.

(e) Set up the equilibrant force needed to maintain the center ring in equilibrium as in Procedure #3 and test the system for equilibrium.

(f) Record all the values in Table #4.

7. Resolve each of the three forces F_3, F_4, and F_5 (from Procedure #6 above) into *x-components* and y-components using trigonometry. Calculate the rectangular components of the resultant force as the algebraic sum of the *x-components* (ΣF_x) and *y-components* (ΣF_y) of the three forces. Find the magnitude of the resultant using Pythagorean theorem and the direction from the arc tangent (tan^{-1}) of the components. Record these results in Table #4.

Table #4:

Mass (kg)	Force	Measured Direction	x-component (N)	y-component (N)
0.100	F_3	30°		
0.200	F_4	120°		
0.150	F_5	220°		
	Equilibrant, F_E			
			ΣF_x =	ΣF_y =
(Graphical) Measured Resultant force, F_R =				
(Graphical) Measured Direction of Resultant, θ =				
(Analytical) Calculated Resultant force, $F_R = \sqrt{[(\Sigma F_x)^2 + (\Sigma F_y)^2]}$ =				
(Analytical) Calculated Direction of Resultant, θ = arc tan[$(\Sigma F_y)/(\Sigma F_x)$] =				

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CALCULATIONS

1. In Table #1, calculate and record the required magnitudes and directions of the force vectors, F_1, F_2, equilibrant force, (F_E), and the resultant (F_R).
2. Compare the calculated resultant (F_R) in Procedure #4 to the value of the equilibrant (F_E) obtained in Procedure #3 above. Compute the percent difference between these two results recorded in Table #1.
3. In Table #2, using the *law of cosines* and the *law of sines,* calculate and record the required magnitude and direction of the resultant (F_R).
4. Compare the calculated resultant (F_R) from Procedure #4 (recorded in Table #2) to the value of the equilibrant (F_E) obtained in Procedure #3 (as recorded in Table #1). Compute the percent difference between the magnitudes of these two forces.
5. In Table #3, calculate and record the required magnitudes and directions of the components of the force, F_x, F_y, the equilibrant force (F_E), and the force (F).
6. Compare the experimental value of the force (F) in Table #3 to the value of the same force calculated analytically. Compute the percent error between the magnitudes of these two results taking the analytical value as the theoretical.
7. Based on the data and results recorded in Table #4, compare the experimental value of the resultant force (obtained from the measurements) with the calculated value (by resolving the vectors into their components analytically). Compute the percent error between these experimental and theoretical values.

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QUESTIONS ______________________________________

1. Compare the results of the resultant vectors in this exercise determined from experimental, graphical, and analytical methods. Considering the analytical method to yield the true resultant vector in each case, is the experimental method more accurate than the graphical method? Justify your answer.
2. List possible sources of error in the experimental methods of determining the resultant force using the force table.
3. In Calculation #4, how should the calculated resultant (F_R) from Procedure #4 compare in magnitude and direction to the value of the equilibrant (F_E) obtained in Procedure #3?
4. The force exerted by gravity on each mass (m) points vertically downward and is equal in magnitude to mg, where the acceleration due to gravity $g = 9.8\ m/s^2$. Considering that the force exerted on the center ring on the force table is in a horizontal plane, explain the function of the pulleys.