Name ________________________________ Group # _________ Date _____________
Partners: ____

PHYS-2212 Laboratory

Electrostatic Fields and Mapping of Equipotential Lines

Objectives

To investigate the lines of equal electrostatic potential (*equipotential lines*) around different configurations of oppositely charged electrodes and from these equipotential lines construct the electric fields for each electrode configuration.

Introduction

An electric field exists in any region of space where electric forces are exerted on any electric charges that may be present. The value of the electric field (E) at every point within the space is determined as the force (F) exerted on a (standardized unit, positive) test charge (q_0) at that specific point. The magnitude of the electric field vector is defined as the electric force exerted on a unit (+1 C) test charge:

$$\vec{E} = \frac{\vec{F}}{q_0}$$

Eqn. 1

The direction of this electric field vector is the direction of the force on the positive test charge placed at the point inside the electric field. Under the action of the electric force, the path traversed by a "free" test charge in an electric field as it moves from one point to another is defined as a line of force. This concept of lines of force is useful in visualizing the magnitude and direction of electric (and magnetic) fields. The direction of the electric line of force at any point is the direction of the electric field at that point.

When the test charge moves from one point (A) to another point (B) inside an electric field, work is done by that electric field. The electric field due to stationary (static) electric charges is a conservative field and thus the total energy is conserved such that:

$\Delta K = -\Delta U.$

Hence the net work done ($Wnet$) by the electric field in moving a charge from point A to point B results in a change in the potential energy (ΔU) of the charge. Recall that from the work-energy theorem, $Wnet = -\Delta U$.

The change in potential energy (ΔU) per unit of charge moved is defined as the electrostatic potential difference (ΔV) or the voltage between those two points in the electric field:

$$\Delta V = \frac{\Delta U}{q_0}$$

Eqn. 2

When the electric force ($F = Eq_0$) is exerted in moving the charge q_0 from point A to point B through displacement Δs, the work done on the charge is given as:

$W = q_0 E. \Delta s = - \Delta U.$

Therefore, if the electrostatic potential difference (ΔV) is measured between two points separated by displacement Δs, the electric field is given as:

$$E = -\frac{\Delta V}{\Delta s}$$

Eqn. 3

However, in the limit as the displacement becomes smaller ($\Delta s \rightarrow 0$) the change in potential energy per unit charge in moving the test charge from point A to point B, i.e. the electrostatic potential difference, is then

$$\Delta V = -\int_A^B E \cdot ds$$

Eqn. 4

Consider two electrodes that are separated by some distance Δs and are connected to the positive and negative terminals of the power supply. The dc power supply maintains a preferred voltage constant between the two electrodes constant. The electrode with the negative charge is arbitrarily assumed to be at zero potential (0 V) and measurements of the potential are made relative to this electrode. Suppose the electrode with the positive charge is set at a positive potential of +15 V. Then, within the space surrounding these electrodes, there must be points that are at the same potential, for example at +10 V. Such points mapped at the same potential form an equipotential surface in a three-dimensional space. However, in two dimensions, as with the electrodes drawn on paper, these points of equal potential will be equipotential lines. These equipotential lines represent the distribution of the electrostatic potential in an electric field, in two dimensions. Since the potential is constant along the equipotential line ($\Delta V = 0$), there is no change in potential difference and hence no work done in moving a test charge along the equipotential line or an equipotential surface. The equipotential lines or surfaces through any point are perpendicular to the direction of the electric field at that point inside the electric field. Therefore, experimentally, the directions of the electric field lines will be constructed from the mapped equipotential lines.

Apparatus

Sheets of high-resistance conductive paper with different electrode configurations (symmetric and non-symmetric), low-voltage dc power supply, field-mapping cork board with probes and metal push pins, connecting wires, high impedance electronic voltmeter (preferably a digital multimeter DMM), metric scale, drawing compass, masking tape, and sheets of 1-cm-by-1-cm graph paper.

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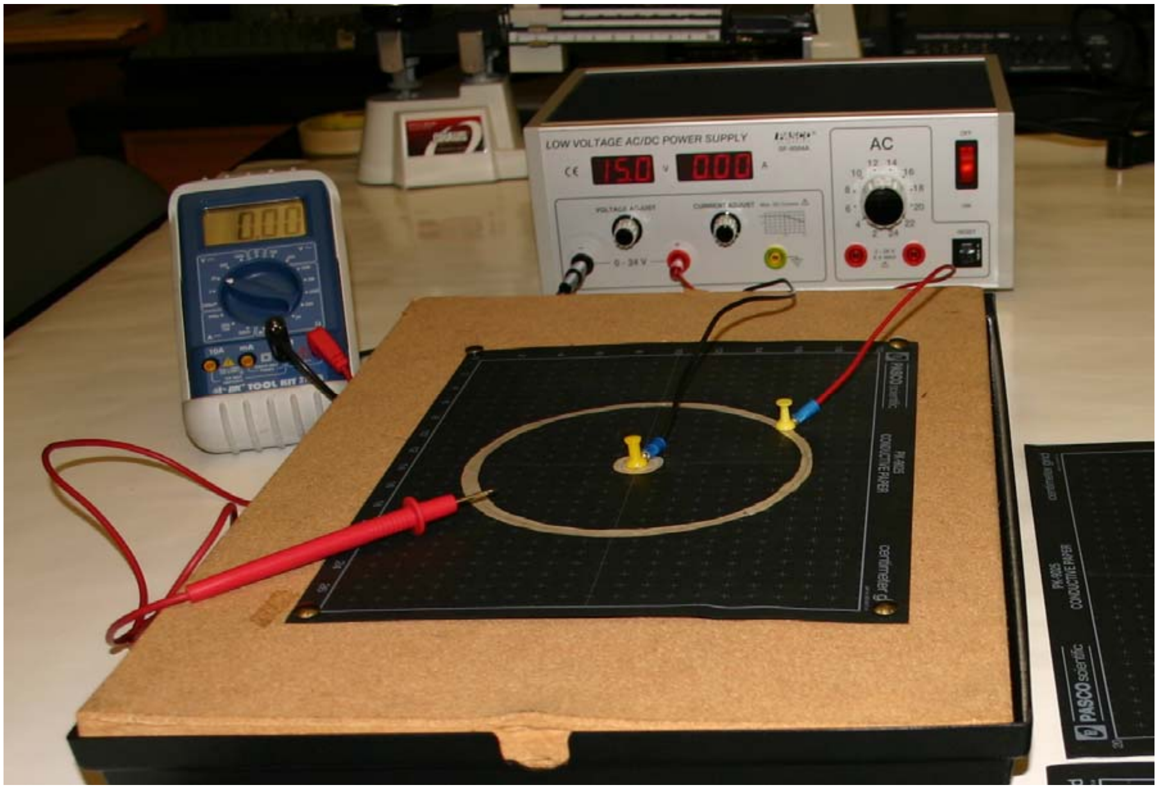

Figure 1: The Electric Field Mapping Setup

Experimental Procedure

1. Assemble the electric field mapping apparatus as shown in the photograph, Figure 1. For each electrode configuration provided, place the conductive sheet on the cork board and secure it firmly on the board with thumb tacks placed at its four corners. Each carbon-impregnated conductive sheet is slightly conducting and is marked with 1-cm-x-1-cm grids across its surface. For convenience and time considerations, an electrode configuration has already been drawn on each conductive sheet using silver conducting paint.
2. For each electrode configuration, construct identical dimensions of the electrodes on a separate graph coordinate paper, drawn to scale. Be sure to properly align the coordinate system of the graph paper to match that of the electrodes to be mapped on the conductive sheet.
3. Connect the DMM, as a voltmeter, to adjust the dc power supply to a voltage output of 15 V. Switch off the power supply and disconnect the DMM from the power supply. Do not change this voltage setting for the rest of the experiment.
4. Firmly press a metal push pin into each silver electrode to ensure good electrical contact with a flat end of a connecting wire from each terminal of the power supply. When the

two electrodes on the conducting paper are connected to the dc power supply, an electric field is established between the electrodes.

5. Connect the negative terminal (COM socket) of the digital voltmeter (DMM) to the negative terminal (0 *V*) of the power supply. Recall that one of the electrodes is also connected to the same negative terminal of the power supply. Thus, the electrode will be considered at *zero potential* (0 *V*)

6. The red (pointed) probe that is connected to the "*V*Ω" (positive) terminal of the DMM will be used as the free "test probe" for mapping the potentials at different points on the conductive paper by touching the probe at points on the paper.

7. **STOP**!!! Ask your lab instructor to inspect your setup. With correct wiring, the instructor will touch the electrode connected to the negative terminal of the power supply with the free test probe to check the zero potential.

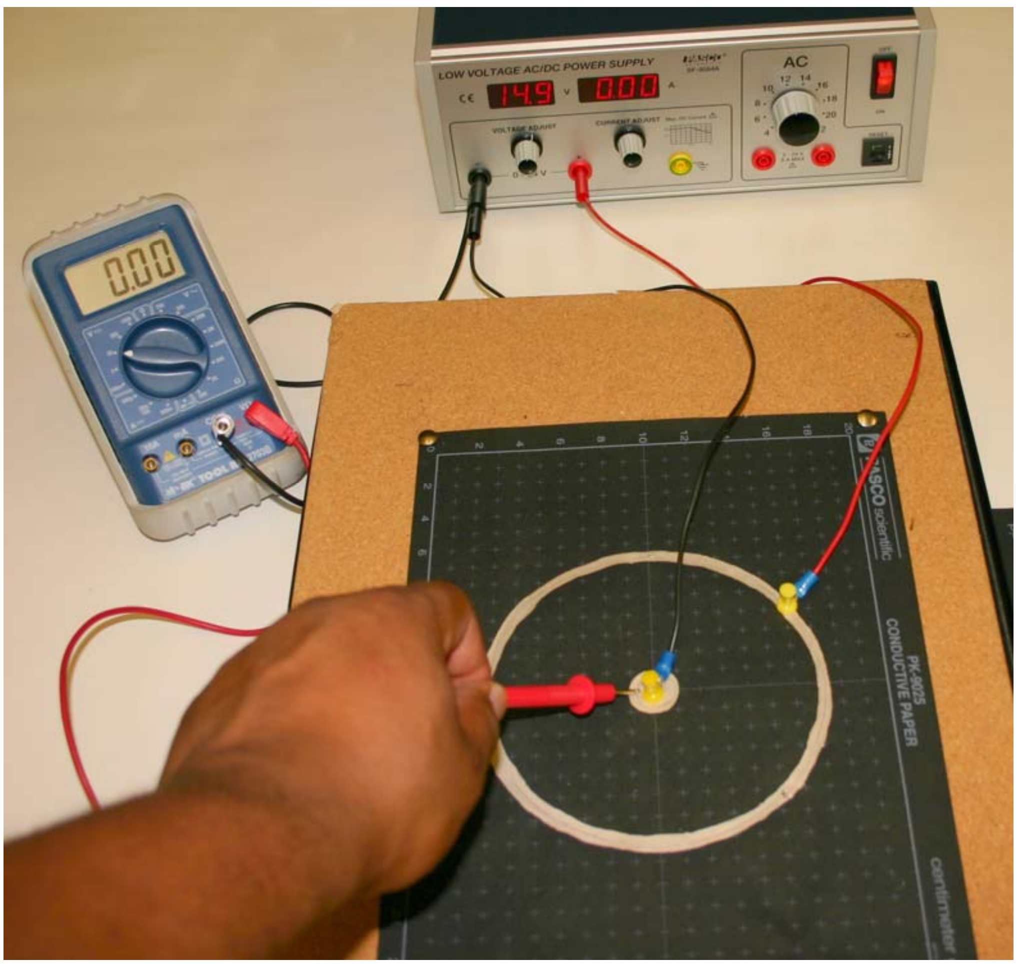

Figure 2: Electric Field Mapping - Checking the Zero Potential

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8. Then the probe is pressed on the other (positive) electrode to confirm a potential of 15 V. It is generally expected that this voltage reading will fall within ±1% of the preset voltage of the power supply. Otherwise, you should check your connections. After your instructor has inspected the wiring for proper conductivity and approved your setup, you are ready to map the equipotential lines between the electrodes.

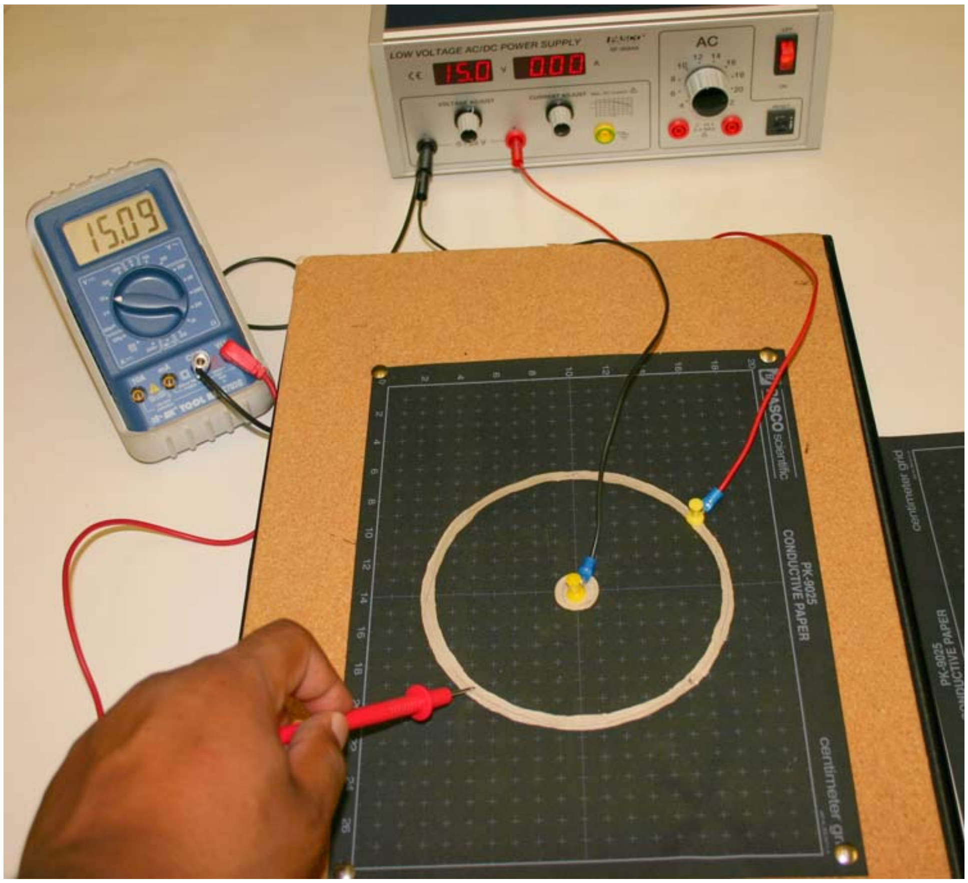

Figure 3: Electric Field Mapping - Checking the Maximum Potential

9. Slowly move the test probe between the electrodes on the conductive paper to locate the point where the voltmeter (DMM) indicates an electrostatic potential reading of 3.0 V. Lightly slide the probe about one centimeter to the right or left of the previous point and locate a series of points at the same voltage reading (3.0 V). *Do not spend too much time in an attempt to locate these points of equal potential to high accuracy!*

10. Plot the *x*- and *y*-coordinates of these equipotential points on the respective graph (coordinate) sheet prepared for that electrode configuration. Connect these points with best smooth curve or line that represents these points. For each potential, obtain just enough points necessary to define the shape of the *equipotential line*. Label this line as the 3-V equipotential line.

11. Repeat the procedure described in steps 8 – 10 above to map the 6-V, 9-V, and 12-V equipotential lines for each electrode configuration.

12. Following the Procedure steps 8 – 11 above, begin with the parallel-electrode configuration (*a*) and plot four equipotential lines on the graph sheet for 3-V, 6-V, 9-V, and 12-V potentials respectively. It does not matter which electrode is connected to zero potential.

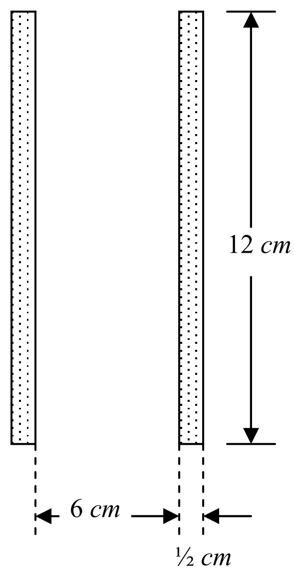

Figure 4a: Two parallel electrode configuration

13. Next, replace the parallel-electrode paper with the concentric-circle configuration (*b*) and repeat the Procedure steps 8 - 11. However, set the central circular electrode at zero potential (0*V*) by connecting it to the negative terminal of the power supply. The outer 7-cm circular electrode should be connected to the positive terminal of the power supply, placing it at a potential of +15 V. Plot four equipotentials (circular lines) as required in step 12.

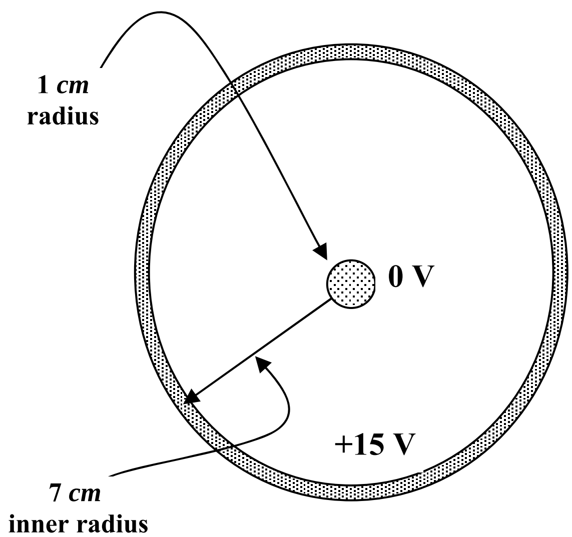

Figure 4b: Concentric circle electrode configuration

14. Replace the concentric-circle electrode configuration with the non-symmetric circular-and-linear electrode configuration (*c*). Repeat the procedure steps for the previous electrode configurations and plot corresponding four equipotential lines. Connect the linear electrode as the zero potential and the circular electrode at a potential of +15 V.

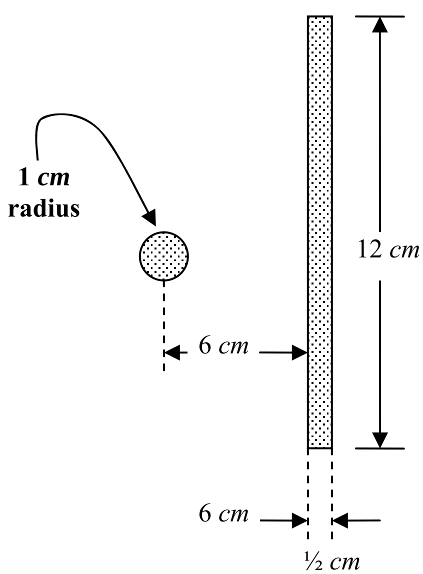

Figure 4c: Circular and Linear electrode configuration

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Data ____

15. **Parallel-Electrode Configuration (*a*)**:
For the parallel electrodes, measure the distances separating adjacent equipotential lines and record these values in the Table 1.

Table 1: Parallel-Electrode Configuration data

Adjacent Equipotential pair (ΔV)	**Distance between equipotential lines (Δx)**
0-3 V	
3-6 V	
6-9 V	
9-12 V	
12-15 V	
Average distance:	

16. **Concentric-Circle Electrode Configuration (*b*)**:
Read the corresponding radial distances of the equipotential circles from the graph sheet for the concentric-circle configuration and record these values in Table 2.

Table 2: Concentric-Circle Electrode Configuration data

Equipotential (V) volts	**0-V**	**3-V**	**6-V**	**9-V**	**12-V**	**15-V**
Radius (r) cm	**1.0**					**7.0**

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17. **Circular-Linear Electrode Configuration (c):**
Measure the corresponding radial distances of the equipotential curves from the graph sheet for this configuration, along the path from the circular electrode and perpendicular to the linear electrode and record these values in Table 3.

Table 3: Circular-Linear Electrode Configuration data

Equipotential (V) volts	**0-V**	**3-V**	**6-V**	**9-V**	**12-V**	**15-V**
Distance along the shortest line between the electrodes (cm)						

Analysis of Data ____

18. Recall that the directions of the electric field lines are everywhere perpendicular to the equipotential surfaces or lines. For each electrode configuration, use the mapped equipotential lines to construct an arbitrary number of smooth *electric field lines* that describe the electric field between the electrodes. *Place arrows to specify the directions of these field lines* from positive to negative charge. In order to distinguish the electric field lines from the equipotential lines, draw these electric field lines in a different color or with dotted lines. Submit all graph sheets.

19. According to the theory, within the small voltage interval between two oppositely-charged parallel electrodes, there exists a uniform electric field, E. Therefore, for an electrostatic potential difference (ΔV) with displacement Δx between the equipotential lines, the electric field is given as:

$$E = -\frac{\Delta V}{\Delta x}$$

From the data Table 1, calculate the average value of the electric field between the parallel electrodes and explain whether this average value of the electric field is approximately constant within experimental uncertainty?

E = ________________________ V/m

20. Using data Table 2, plot the radial distances (r) recorded (as ordinates) against the matching equipotentials (as abscissa) on a semi-logarithmic graph. Note that based on the experimental setup, there is already a zero-volt equipotential at a radius of 1 cm and a 15-V equipotential at a radius of 7 cm. Draw the best straight line through these data points. Calculate the slope of this semi-logarithmic graph of r vs. V.

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Slope of semi-log graph = ______________________ cm/V

21. According to theory, it can be shown using **Eqn. 4** that in the space between two oppositely-charged concentric cylinders (in three dimensions), equipotentials have radii (r) that increase logarithmically with voltage (V) according to the equation: $V(r) = \frac{Q}{2\pi\varepsilon_o l}\ln\left(\frac{r}{a}\right)$,
where an electric charge Q is uniformly distributed on the surface of the inner cylinder of radius a and length l. Note that based on the dimensions of the electrodes, a = 1 cm. Comment on the significance of the slope of the semi-log graph of the radii r vs. V and compare these results to the theoretical model.

22. Using the data obtained for the concentric circle electrodes in Table 2 and the corresponding graph sheet, measure the radial distances (r) to the midpoint between each pair of adjacent equipotential circles. Calculate and record the reciprocal of each of these radial distances (*1/r*). Enter these results in the pertinent columns of Table 4 below.

Table 4: Electric field values for the concentric-circle electrode configuration

Adjacent Equipotential pair *(ΔV)*	**Distance between equipotentials** *(Δr)*	**Radial distance to midpoint between equipotentials** *(r)*	**Electric Field at the midpoint** *(E = ΔV/ Δr)*	**Reciprocal of radius to midpoint between equipotentials** *(1/r)*
0-3 V				
3-6 V				
6-9 V				
9-12 V				
12-15 V				

23. From data in Table 4, plot a graph of *E* against *1/r* for the concentric-circle configuration. Draw a regression line through your points and calculate its slope.

Slope of = _______________________ V

24. According to theory, the electric field for a charged conducting solid cylinder (in three dimensions) is inversely proportional to the radial distance from the cylinder according to the equation, $E = \frac{Q}{2\pi\varepsilon_0 l} \cdot \left(\frac{1}{r}\right)$, where an electric charge *Q* is uniformly distributed on the surface of the cylinder of length *l*.. Therefore, based on the *E* vs. *1/r* graph and its regression coefficient, comment on the significance of the graph and its calculated slope when compared to the theoretical model.

25. The equipotential curves and the electric field lines of the circular-linear electrode configuration represent the two-dimensional scale model of certain electrostatic “shocking” real world observations. Can you think of two possible examples?

26. Examine your equipotential field maps and the matching electric field patterns for all three electrode configurations and discuss any observed relationship between the regions where the electric field lines are closely spaced and regions of closely-spaced equipotential curves.