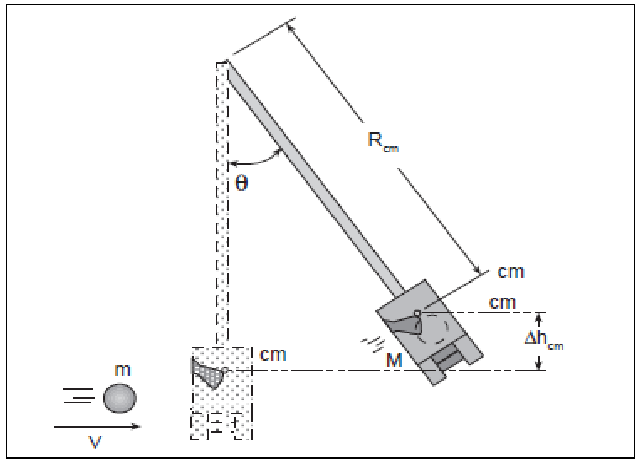
Ballistic Pendulum

Equipment Needed	Qty
ballistic pendulum	1
ruler	1
2 meter stick	1
plum bob	1
carbon paper	1
balance	1



This exercise illustrates the concepts of **conservation of energy** and **conservation of momentum**, and uses a type of device (the **ballistic pendulum**) originally invented for determining the speed of a bullet.

In our version, a steel ball is shot from a spring-powered projectile launcher and lodges itself in a motionless pendulum (figure). The pendulum and ball swing up due to the ball's momentum, and the height Δh_{CM} to which they swing is used to determine the ball's speed prior to striking the pendulum.

Because of the conservation of momentum, the ball's momentum prior to striking the pendulum is equal to the momentum of the ball and pendulum after contact.

$$p_1 = p_2$$

$$mv_1 = (m + M)v_2 \tag{1}$$

p_1	momentum of ball before contact
p_2	momentum of ball/pendulum after contact
m	mass of ball
v_1	speed of ball before contact
M	mass of pendulum
v_2	speed of ball/pendulum after contact

In this way, the speed of the ball prior to contact (v_1) can be determined from the speed of the ball/pendulum after contact (v_2). Moreover, v_2 can be determined from Δh_{CM}, the height increase of the ball/pendulum. So then, v_1 can be determined from Δh_{CM}.

The conservation of energy is used to relate v_2 and Δh_{CM} . The kinetic energy of the moving pendulum/ball is converted to potential energy as their center of mass rises. So then, their speed after contact is related to the height to which they rise by:

$$KE_2 = PE_3$$

$$\tfrac{1}{2}(m + M)v_v^2 = (m + M)g\Delta h_{CM}$$

$$v_2 = \sqrt{2g\Delta h_{CM}} \quad (2)$$

KE_2	Kinetic energy of ball and pendulum after contact
PE_3	Potential energy of ball and pendulum at peak of swing.
Δh_{CM}	Distance that ball/pendulum center of mass rises

In our setup, Δh_{CM} is determined from θ, the angle of the pendulum at maximum swing (figure). Some trigonometry shows that

$$\Delta h_{CM} = R_{CM}(1 - cos\theta) \quad (3)$$

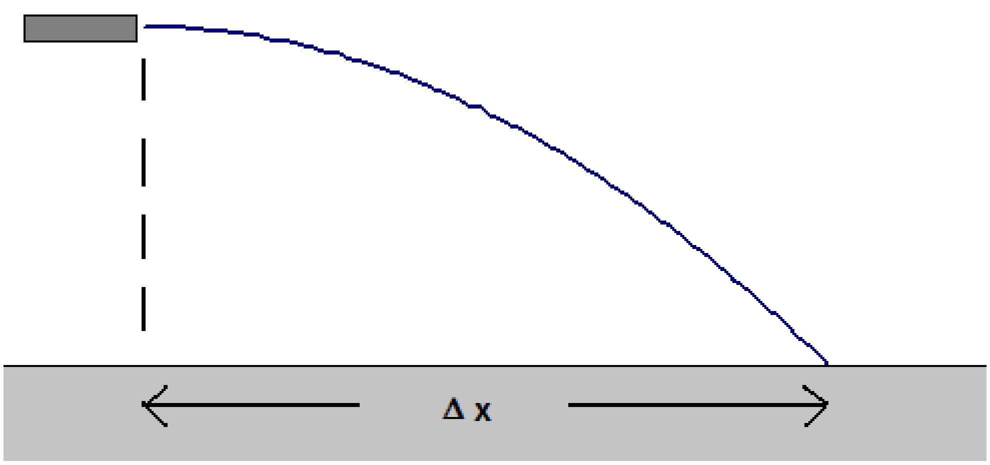
Where R_{CM} measures from the pendulum pivot to the center of mass. When measuring this be sure that the ball is in the pendulum and find the balance point using the fulcrum.

Procedure:

The following steps can be used to determine v_1 from θ (as always, remember to use SI units for all quantities).

1. Measure and record the following: R_{CM} = ________ m = ________ M = ________
2. Fire the apparatus 10 times, recording θ each time and find their average. θ_{avg} = ________
3. Use equation 3 to calculate Δh_{CM} from the average θ. Δh_{CM} = ________
4. Use equation 2 to calculate v_2 from Δh_{CM}. v_2 = ________
5. Use equation 1 to calculate v_1 from v_2. v_1 = ________

With this prediction of the ball's speed out of the gun, you can now predict using two dimensional kinematics how far it will travel before hitting the floor (below). Lock the pendulum in the up position (so that it is horizontal) so that you can fire the ball onto the floor.



1. Measure Δy, the vertical distance that the ball will drop to hit the floor after firing (there is a figure of the ball on the side of the gun to indicate its position at launch). You can move the launcher to the edge of the table and measure directly to the floor. Δy = ________
2. Use the equations of two dimensional kinematics (below) to find Δx, the distance that the ball should move horizontally before hitting the floor. Δx_{pred} = ________

3. Place the apparatus so that the ball will fire out over the floor. Then find the position on the floow that you predict the ball will land. Keep in mind that the ball will start its flight not at the edge of the table, but directly underneath the figure of the ball on the gun. You can use the plum bob to find the point on the floor directly below the edge of the table and use a ruler and 2 meter stick to find the point in question.
4. Place a target sheet on the floor such that the line on the sheet marks the landing point. On top of that, place a piece of carbon paper carbon down. When the ball strikes this, it will make a dot on the paper beneath. Determine the actual Δx. $\Delta x_{exp} =$ _______

$$\Delta v_x = a_x \Delta t \qquad \Delta v_y = a_y \Delta t$$
$$\Delta x = v_{ox}\Delta t + {}^1\!/_2 a_x (\Delta t)^2 \qquad \Delta y = v_{oy}\Delta t + {}^1\!/_2 a_y (\Delta t)^2$$
$$v_{fx}^2 = v_{ox}^2 + 2a_x \Delta x \qquad v_{fy}^2 = v_{oy}^2 + 2a_y \Delta y$$

Questions

1. Find the percent difference between the predicted and the measured Δx. $\% \, diff =$ _______
2. Determine KE_1, the ball's kinetic energy prior to contact with the pendulum, as well as KE_2, the kinetic energy of the ball/pendulum after contact. $KE_1 =$ _______ $KE_2 =$ _______
3. The KE_2 (of the ball/pendulum) should be smaller than KE_1 (of the ball prior contact). What percentage of KE_1 is KE_2? _______
4. Since energy is a conserved quantity, all the energy in the system prior to contact still remains after contact. What form does the missing energy take after contact? _____________________
5. How much energy is this in Joules? $Q =$ _______
6. The impact of the ball causes the pendulum to exert a forward impulse on the pivot on which it swings. Thus not all of the ball's momentum is in the ball/pendulum after contact, a fact that we have not taken into consideration in our calculations. As a result of this oversight, is our estimate of the ball's velocity too high or too low? _____________________
7. It turns out that $KE_2/KE_1 = m/(m+M)$. If we had used a plastic ball instead of a steel ball, would this fraction be greater or smaller? _____________________